UNIT 3

SINGLE PHASE TRANSFORMER

4.1 INTRODUCTION

Transformer is a static device which transfers electrical energy from one circuit to another circuit by changing the level of the voltage without changing the frequency. The symbol of the transformer is shown in fig. 8.1.

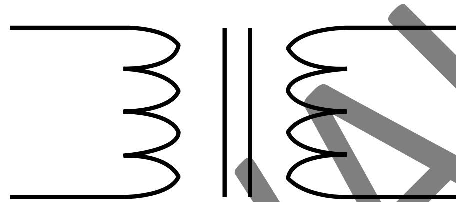


Fig. 8.1

4.2 CONSTRUCTION OF THE TRANSFORMER

Construction of the transformer mainly consists of three parts as shown in fig. 8.2

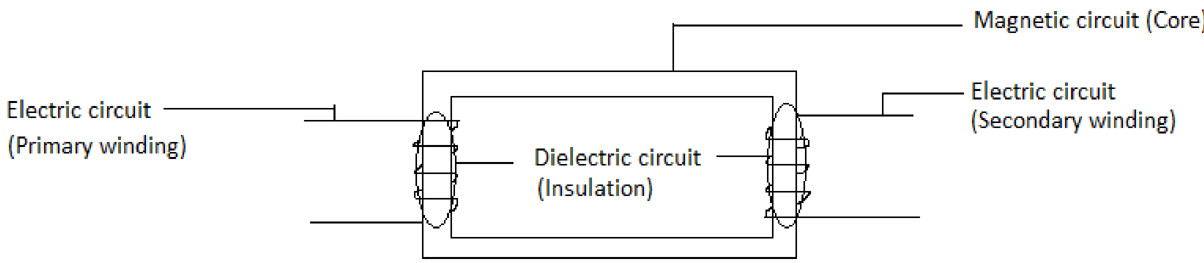


Fig. 8.2

4.2.1 **Magnetic circuit:** The basic function of the magnetic circuit is to provide the path for the flow of flux. The magnetic circuit of the transformer consist a core. The core of the transformer is either square of rectangular. The core is made by the CRGO.

4.2.2 **Electric circuit:** The basic function of the electric circuit is to provide the path for the flow of current. The electric circuit of the transformer consist two winding called primary winding and secondary winding. The winding of the transformer is made by the copper. Both the winding of the transformer consist different number of turns. There is no electrical connection between primary and secondary winding.

4.2.3 **Dielectric circuit:** The basic functions of the dielectric circuit it to insulate the different parts of the transformer to each other. The dielectric circuit of the transformer consist paper, board which is used at the different places of the transformer.

4.3 PRINCIPLE OF THE TRANSFORMER

The principle of the transformer is the faraday law of Electro Magnetic Induction (EMI).According to this law "If there is the rate of change in the flux link with the winding then EMF will induced in the winding". The value of the induced EMF is given by

$$e = -N\frac{d\emptyset}{dt}$$

Negative sign is because of the lan'z law. According to lan'z law this emf oppose to the reason by which it produce.

4.4 WORKING OF THE TRANSFORMER

When an AC supply (V_1) is given to the primary winding of the transformer as shown in fig. 8.3. Then an alternating current (I_1) will start to flow in the primary winding, so an alternating flux (Ø) is set up in the core of the transformer. This alternating flux links with both the winding. Then according to faraday law of Electro Magnetic Induction (EMI) an EMF will induce in both primary (E_1) and secondary (E_2) winding. Now if load is connected to the secondary winding then current will start to flow in the secondary winding (I_2) and we get the voltage (V_2) to the output terminal. So without any electrical connection between the primary and secondary winding energy is transferred from primary winding to secondary winding.

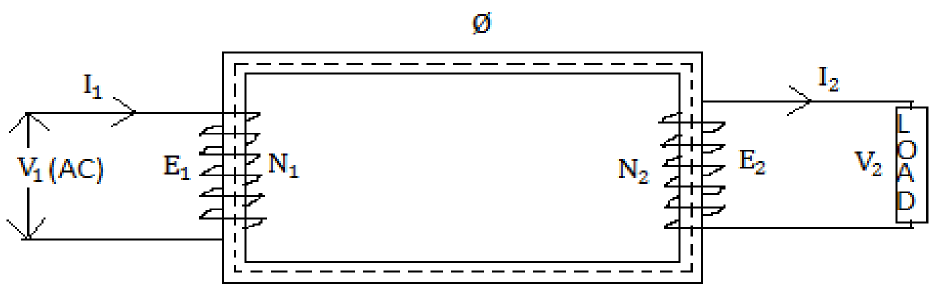


Fig. 8.3

4.5 TRANSFORMER ON DC SUPPLY

The DC supply cannot be use for the transformer because with DC supply the flux produced in the core of the transformer will not vary with time but remains constant, and then faraday law is not applicable. EMF will not induce in the secondary winding. The DC supply never connected to the transformer because it will burn the transformer.

4.6 EMF EQUATION OF THE TRANSFORMER

The parameters of the transformer shown in fig. 8.4 are

V_1 = Supply voltage or primary voltage
V_2 = Output voltage or secondary voltage
I_1 = Supply current or primary current
I_2 = Output current or secondary current
E_1 = Induced EMF in the primary
E_2 = Induced EMF in the secondary
N_1 = Number of turn in the primary
N_2 = Number of turn in the secondary
Ø = Flux in the core

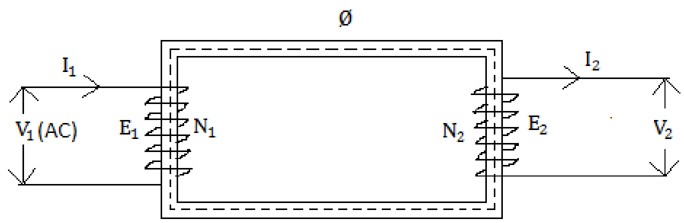


Fig. 8.4

The AC voltage is given to the primary winding of the transformer, so the current flowing in the primary is also AC in the nature. Due to this AC current the flux generated in the core of the transformer is also AC in the nature. This AC flux can be given by the eq.

$$\emptyset = \emptyset_m \sin \omega t \quad \text{..........(1)}$$

According to faraday law of EMI the induced emf is given by

$$e = -N\frac{d\emptyset}{dt}$$

Substitute the value of flux,

$$e = -N\frac{d(\emptyset_m \sin \omega t)}{dt} = -N\omega\emptyset_m \cos \omega t = -N\omega\emptyset_m \sin(90° - \omega t)$$

$$e = N\omega\emptyset_m \sin(\omega t - 90°)$$

This EMF will be maximum when $\sin(\omega t - 90°) = 1$

The value of maximum flux is

$$E_m = N\omega\emptyset_m$$

So the eq. of EMF will be

$$e = E_m \sin(\omega t - 90°) \quad \text{..........(2)}$$

By comparing the eq. (2) and eq. (1) it is clear that the induced emf (both e_1 and e_2) is lagging from the flux by an angle 90°.

The RMS value of induced emf is

$$E = \frac{E_m}{\sqrt{2}} = \frac{N\omega\emptyset_m}{\sqrt{2}} = \frac{N2\pi f\emptyset_m}{\sqrt{2}}$$

$$E = 4.44Nf\emptyset_m$$

The induced emf in the primary winding

$$E_1 = 4.44N_1f\emptyset_m$$

The induced emf in the secondary winding

$$E_2 = 4.44N_2f\emptyset_m$$

From the eq. of E_1 and E_2 it is clear that the difference is only no. of turns. The side which has more no. of turns will get the more emf. According to this transformer are of two types

(1) If $N_1 > N_2$ Then $E_1 > E_2$ It is called step down transformer

(2) If $N_2 > N_1$ Then $E_2 > E_1$ It is called step up transformer

4.7 TRANSFORMATION RATIO OF THE TRANSFORMER

Transformation ratio gives the relation between the primary parameter to the secondary parameter. The transformation ratio (k) is

$$k = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

4.8 IDEAL AND PRACTICAL TRANSFORMER

For understanding the transformer some assumption are made in transformer these assumption are

(1) No Winding resistance: The primary and secondary winding of the transformer have zero resistance

(2) No magnetic leakage: There is no leakage in the flux and all the flux set up in the core.

(3) No iron loss: Hysteresis and eddy current losses in the core are zero.

(4) Zero magnetizing current: zero magnetizing current is required for set up the flux in the core.

The transformer which consist the above assumption are called ideal transformer.

4.9 PHASOR DIAGRAM OF THE TRANSFORMER ON NO LOAD CONDITION

The transformer under no load condition is shown in fig. 8.5

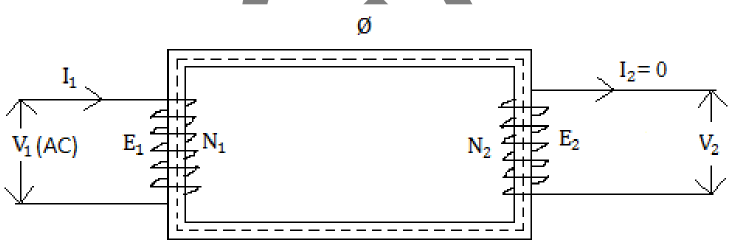


Fig. 8.5

The different parameters are

V_1 = Supply voltage or primary voltage
V_2 = Output voltage or secondary voltage
I_1 = Supply current or primary current
I_2 = Output current or secondary current
I_m = Magnetizing component of no load current
I_a = Active component of no load current
I_0 = No load current
E_1 = Induced emf in the primary
E_2 = Induced emf in the secondary
N_1 = Number of turn in the primary
N_2 = Number of turn in the secondary
$\emptyset$ = Flux in the core
$\emptyset_1$ = Phase angle of the primary winding

$\emptyset_2$ = Phase angle of the secondary winding

No load current $(\mathbf{I_0})$: It is the current which is flowing in the primary winding of the transformer under no load condition. It is the phasor addition of I_m and I_a.

Magnetizing component of current $(\mathbf{I_m})$: It is the reactive component of the no load current. This component is responsible to produce the flux in the core of the transformer so it is in same phase with the flux.

Active component of current $(\mathbf{I_a})$: It is the loss component of the no load current. This component is responsible to supply the losses in the transformer so it is leading from the flux be an angle 90°.

From the phasor diagram shown in fig. 8.6

$$I_0 = \sqrt{I_m^2 + I_a^2}$$

Steps to draw the phasor diagram

(1) Draw Ø: The flux (Ø) is same in the primary and secondary winding so take Ø is the reference for drawing the phasor diagram
(2) Draw I_m: It is in same phase with the flux.
(3) Draw I_a: It is leading from the flux by an angle 90°.
(4) Draw I_0: It is the resultant of the I_m and I_a.
(5) Draw E_1: It is lagging from the flux by an angle 90°.
(6) Draw E_2: It is lagging from the flux by an angle 90°.
(7) Draw V_1: It is equal and opposite to the E_1.
(8) Draw V_2: It is equal and in same phase with to the E_2.

The phasor diagram of the transformer under no load condition is shown in fig. 8.6

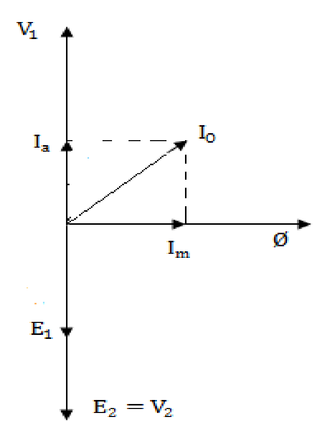


Fig. 8.6

4.10 PHASOR DIAGRAM OF THE TRANSFORMER ON LOADED CONDITION

The transformer under loaded condition is shown in fig. 8.7

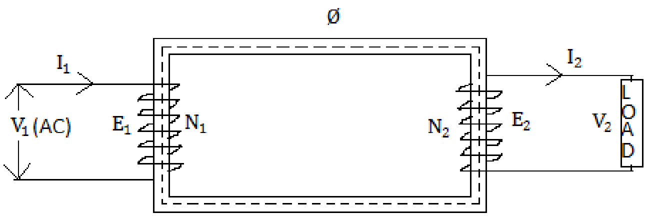


Fig. 8.7

The different parameters are

V_1 = Supply voltage or primary voltage
V_2 = Output voltage or secondary voltage
I_1 = Supply current or primary current
I_2 = Output current or secondary current
I_m = Magnetizing component of no load current
I_a = Active component of no load current
I_0 = No load current
I_1' = Current in the primary winding due to the secondary winding current (I_2)
E_1 = Induced emf in the primary
E_2 = Induced emf in the secondary
N_1 = Number of turn in the primary
N_2 = Number of turn in the secondary
$\emptyset$ = Flux in the core
$\emptyset_1$ = Phase angle of the primary winding
$\emptyset_2$ = Phase angle of the secondary winding

I_1' = When I_2 current is flowing in secondary winding then due to this current a current kI_2 is flowing in the primary winding it is called I_1'.

Steps to draw the phasor diagram

(9) Draw $\emptyset$: The flux ($\emptyset$) is same in the primary and secondary winding so take $\emptyset$ is the reference for drawing the phasor diagram
(10) Draw I_m: It is in same phase with the flux.
(11) Draw I_a: It is leading from the flux by an angle 90°.
(12) Draw I_0: It is the resultant of the I_m and I_a.
(13) Draw E_1: It is lagging from the flux by an angle 90°.
(14) Draw E_2: It is lagging from the flux by an angle 90°.
(15) Draw V_1: It is equal and opposite to the E_1.
(16) Draw V_2: It is equal and in same phase with to the E_2.
(17) Draw I_2: It will depend on the load. The load are of three types
   (i) For R-L load: I_2 is lagging from the V_2 by an angle $\emptyset_2$ as shown in fig. 8.8 (a).

(ii) For R-C load: I_2 is leading from the V_2 by an angle $\emptyset_2$ as shown in fig. 8.8 (b).

(iii) For R load: I_2 is in same phase with V_2 as shown in fig. 8.8 (c).

(18) Draw I_1': It is just opposite to the I_2.

(19) Draw I_1: It is the resultant of the I_0 and I_1' .

The phasor diagram of the practical transformer under different loaded condition is shown in fig. 8.8

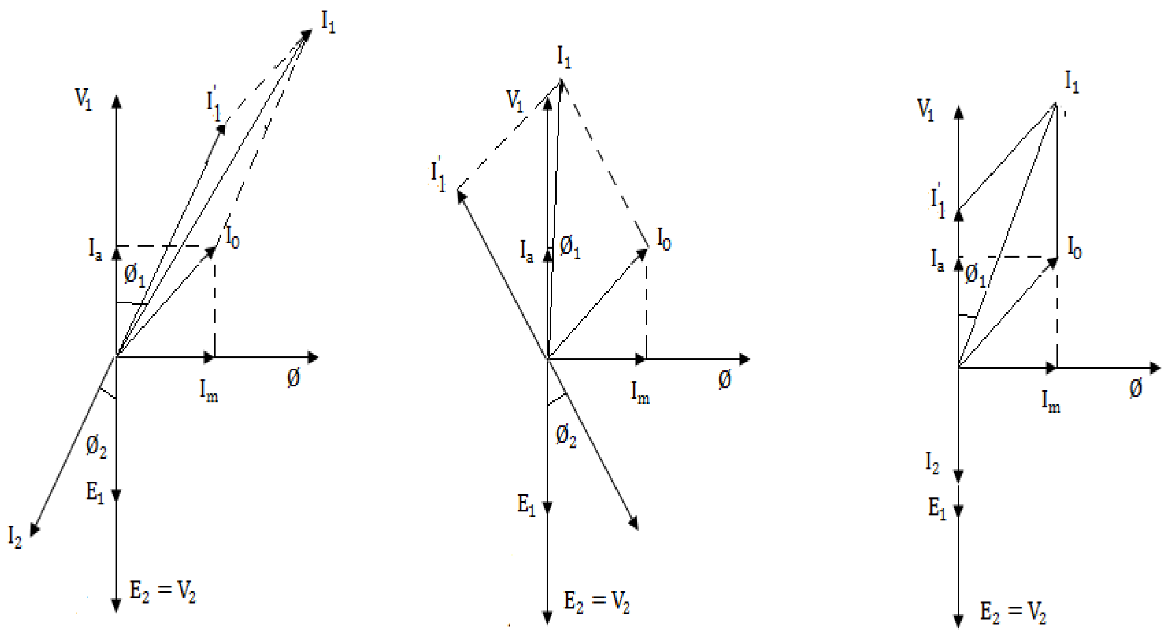


Fig. 8.8 (a) **Fig 8.8 (b)** **Fig. 8.8 (c)**

4.11 RULE FOR SHIFTING THE IMPEDANCE OF THE TRANSFORMER

We know

$$k = \frac{V_2}{V_1} \text{ and } k = \frac{I_1}{I_2}$$

By multiplying both, we get

$$k^2 = \frac{V_2}{V_1} \times \frac{I_1}{I_2} = \frac{Z_2}{Z_1}$$

..........(1)

Where

$Z_2 = \frac{V_2}{I_2}$ and $Z_1 = \frac{V_1}{I_1}$

According to eq. (1)

$Z_1 = \frac{Z_2}{k^2}$, for transfering the secondary impedance$(Z_2)$to primary we have to divide $Z_2$by k^2.

$Z_2 = k^2 Z_1$, for transfering the primary impedance$(Z_1)$to secondary we have to miltiply $Z_1$by k^2.

<u>Note:</u> The same rule is applicable for resistance and reactance.

4.12 EQUIVALENT CIRCUIT OF THE TRANSFORMER

Equivalent circuit of the transformer is very useful for different calculation of the transformer. The equivalent circuit of the transformer is shown in fig. 8.9.

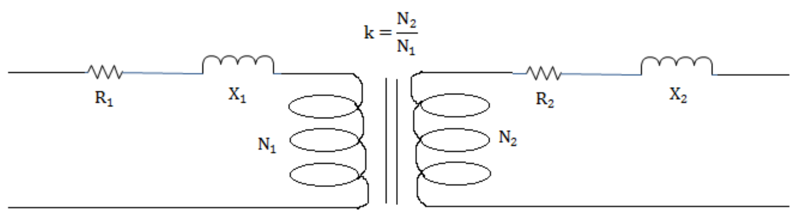


Fig. 8.9

Where

R_1 = The resistance of the primary winding

X_1 = The reactance of the primary winding

R_2 = The resistance of the secondary winding

X_2 = The reactance of the secondary winding

k = Transformation ratio

This equivalent circuit consist two winding for simplification we can convert these two winding into one winding. According to these equivalent circuit are of two types

4.12.1 **Equivalent circuit referred to primary:** In this case parameter of the secondary side is shifted to the primary side as shown in fig. 8.10.We know when the parameter of the secondary is shifted to primary they will be divided by k^2.

a

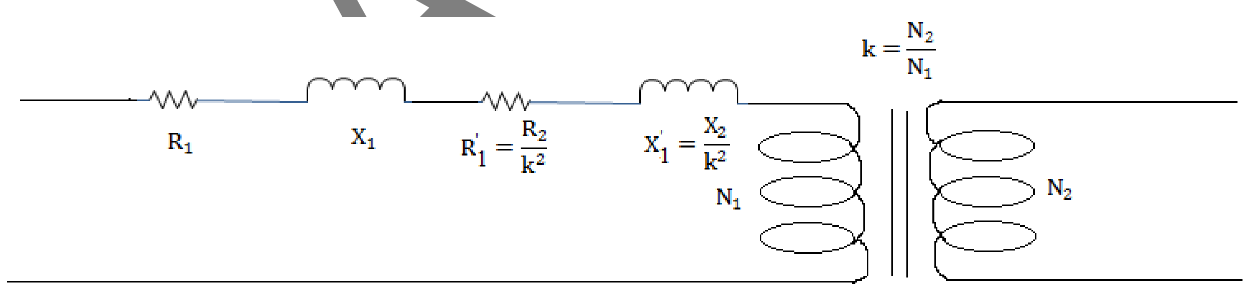


Fig. 8.10

Where

R_1' = It is resistance of the primary winding due to the secondary winding resistance R_2.

X_1' = It is reactance of the primary winding due to the secondary winding reactance X_2.

Equivalent resistance of the transformer referred to primary (R_{01}): It is the total resistance of the primary when secondary referred to primary

$$R_{01} = R_1 + R_1'$$

$$R_{01} = R_1 + \frac{R_2}{k^2}$$

Equivalent reactance of the transformer referred to primary (X_{01}): It is the total reactance of the primary when secondary referred to primary

$$X_{01} = X_1 + X_1'$$
$$X_{01} = X_1 + \frac{X_2}{k^2}$$

Equivalent impedance of the transformer referred to primary (Z_{01}): It is the total impedance of the primary when secondary referred to primary

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

4.12.2 **Equivalent circuit referred to secondary:** In this case parameter of the primary side is shifted to the secondary side as shown in fig.8.11. We know when the parameter of the primary is shifted to secondary they will be multiply by k^2.

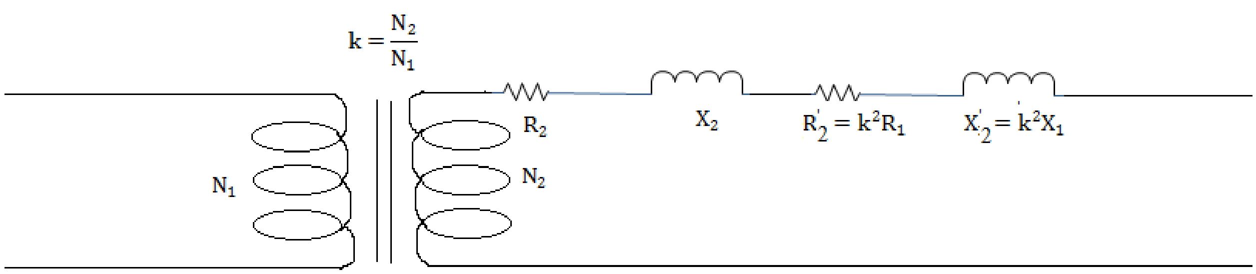


Fig. 8.11

Where

R_2' = It is resistance of the secondary winding due to the primary winding resistance R_1.

X_2' = It is reactance of the secondary winding due to the primary winding reactance X_1.

Equivalent resistance of the transformer referred to secondary (R_{02}): It is the total resistance of the secondary when primary referred to secondary

$$R_{02} = R_2' + R_2$$
$$R_{02} = k^2 R_1 + R_2$$

Equivalent reactance of the transformer referred to secondary (X_{02}): It is the total reactance of the secondary when primary referred to secondary

$$X_{02} = X_2' + X_2$$
$$X_{02} = k^2 X_1 + X_2$$

Equivalent impedance of the transformer referred to secondary (Z_{02}): It is the total impedance of the secondary when primary referred to secondary

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$

4.13 LOSSES IN THE TRANSFORMER

Transformer losses are of two types

4.13.1 **Core loss (Iron loss):** The losses in the core of the transformer are called core loss. The core is generally made by the iron so it is also called iron loss. It is denoted by (P_i). Core loss are of two types

(i) **Hysteresis loss:** The primary winding of the transformer is supply by alternating current then the flux in the core is also alternating in the nature. This alternating flux magnetizes and demagnetizes the core in each cycle. Due to this there is the loss of energy by which core is heated. This energy loss is called the hysteresis loss. It is denoted by P_h.

$$P_h = K_h B_m^{1.6} f V \quad \text{watt}$$

Where

K_h = Hysteresis loss constant

B_m= Maximum flux density

f = Frequency

V = Volume of the core

(ii) **Eddy current loss:** When the primary winding of the transformer is supplied with alternating current, which is wound on the core, then according to faraday law a current is induced in the core. It is a circulating current on the surface of the core called eddy current. Due to this current the loss of energy by which core is heated. This energy loss is called the eddy current loss. It is denoted by P_e.

$$P_e = K_e B_m^2 f^2 t^2 \quad \text{watt}$$

Where

K_e = Eddy current loss constant

B_m= Maximum flux density

f = Frequency

t = Thickness of the core

Total core loss = Hysteresis loss + Eddy current loss

$$P_i = P_h + P_e$$

Note: Core loss is also called constant loss

4.13.2 **Winding loss (Copper loss):** The losses in the winding of the transformer are called winding loss. The winding is generally made by the copper so it is also called copper loss. It is denoted by (P_c). Due to current (I) and resistance (R) of the winding the power loss I^2R happen in the winding is called copper loss. The transformer consist of two winding primary and secondary, so total copper loss

P_c = copper loss in the primary winding + copper loss in the secondary winding

$$P_c = I_1^2 R_1 + I_2^2 R_2$$

Total copper loss when transformer referred to primary

$$P_c = I_1^2 R_{01}$$

Total copper loss when transformer referred to secondary

$$P_c = I_2^2 R_{02}$$

Total loss = Iron loss + Copper loss

$$P = P_i + P_c$$

4.14 VOLTAGE REGULATION

It is defined as the ratio of change in secondary terminal voltage from no load to full load to the secondary no load voltage.

If V_2 is the secondary voltage at full load and E_2 is the secondary voltage at no load then percentage voltage regulation is given by

$$\text{Voltage Regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

4.15 EFFICIENCY

Efficiency is defined as the ratio of output power to the input power. It is denoted by η.

$$\eta = \frac{\text{output power}}{\text{input power}}$$

$$\text{Output power} = V_2 I_2 \cos\emptyset_2$$

Input power = output power + losses

Input power = output power + iron loss + copper loss

$$\text{Input power} = V_2 I_2 \cos\emptyset_2 + P_i + P_c$$

So

$$\eta = \frac{V_2 I_2 \cos\emptyset_2}{V_2 I_2 \cos\emptyset_2 + P_i + P_c}$$

We know

$$P_c = I_2^2 R_{02}$$

So efficiency

$$\eta\% = \frac{V_2 I_2 \cos\emptyset_2}{V_2 I_2 \cos\emptyset_2 + P_i + I_2^2 R_{02}} \times 100$$

Condition for maximum efficiency:

The maximum efficiency obtained when

$$\frac{d\eta}{dI_2} = 0$$

$$\frac{d}{dI_2}\left(\frac{V_2 I_2 \cos\emptyset_2}{V_2 I_2 \cos\emptyset_2 + P_i + I_2^2 R_{02}}\right) = 0$$

$$\frac{(V_2 I_2 \cos\emptyset_2 + P_i + I_2^2 R_{02})(V_2 \cos\emptyset_2) - (V_2 I_2 \cos\emptyset_2)(V_2 \cos\emptyset_2 + 2I_2 R_{02})}{(V_2 I_2 \cos\emptyset_2 + P_i + I_2^2 R_{02})^2} = 0$$

$$(V_2 \cos\emptyset_2)(V_2 I_2 \cos\emptyset_2 + P_i + I_2^2 R_{02} - V_2 I_2 \cos\emptyset_2 - 2I_2^2 R_{02}) = 0$$

$$P_i + I_2^2 R_{02} - 2I_2^2 R_{02} = 0$$

$$P_i - I_2^2 R_{02} = 0$$

$$P_i = P_c$$

$$\text{Iron loss} = \text{Copper loss}$$

Maximum efficiency can be obtained when iron losses and copper losses are equal.

4.16 AUTO TRANSFORMER

An auto transformer is a special type of transformer in which a part of winding is common to both primary and secondary. The operating principle and general construction of the autotransformer is similar to the two winding transformer.

In a two winding transformer the primary and secondary winding are isolated, but in auto transformer the primary and secondary winding are not isolated. In auto transformer the primary and secondary winding are connected to each other.

The auto transformer is of two types

(1) Step up auto transformer: Shown in fig. 8.12 (a).
(2) Step down auto transformer: Shown in fig. 8.12 (b).

Auto transformer is shown in fig.

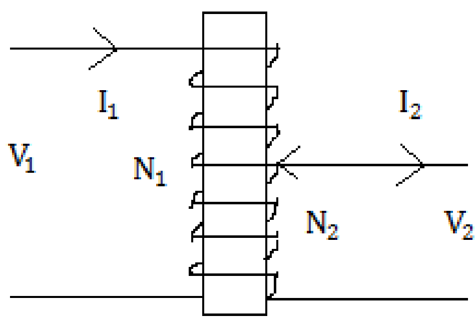


Fig. 8.12 (a)

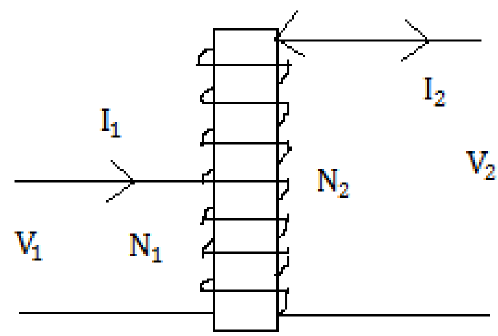


Fig. 8.12 (b)

Advantage of auto transformer: the advantage of auto transformer from the two winding transformer are as follow.

(1) The auto transformer required less copper material.
(2) The auto transformer is smaller in size.
(3) The auto transformer is cheaper.
(4) Due to less copper material, copper losses are reduced.
(5) The efficiency of the auto transformer is higher.

Limitation of the auto transformer:

(1) No electrical separation between primary and secondary which is risky in the case of high voltage.
(2) The short circuit current in auto transformer is very high.

Application of the auto transformer:

(1) It is use as a variac (variable AC) in the laboratory.
(2) It can be use as a regulating transformer.

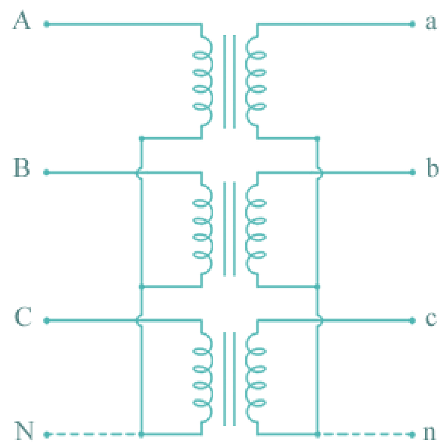
4.17 THREE PHASE TRANSFORMER

The winding of the three phase transformers may be connected either in star or in delta. The primary as well as secondary of the three phase transformer are connected either in star or in delta, So there are four ways of connecting the winding of a 3 phase transformer are, Y-Y, Δ-Δ, Y-Δ, Δ-Y.

(1) Y-Y (Star-Star) connection: It is used for small current, high voltage transformer.

$$\text{Voltage ratio} = \frac{V_{L2}}{V_{L1}} = \frac{\sqrt{3}V_{P2}}{\sqrt{3}V_{P1}} = \frac{N_2}{N_1}$$

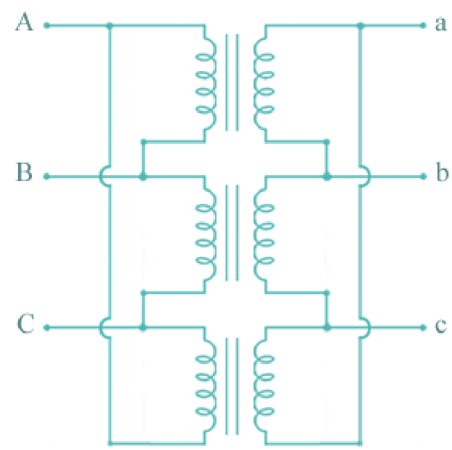
$$\text{Current ratio} = \frac{I_{L2}}{I_{L1}} = \frac{I_{P2}}{I_{P1}} = \frac{N_1}{N_2}$$



(2) Δ-Δ (Delta-Delta) connection: It is used for high current, low voltage transformer.

$$\text{Voltage ratio} = \frac{V_{L2}}{V_{L1}} = \frac{V_{P2}}{V_{P1}} = \frac{N_2}{N_1}$$

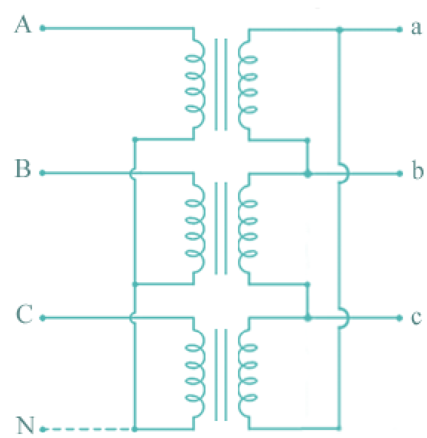
$$\text{Current ratio} = \frac{I_{L2}}{I_{L1}} = \frac{\sqrt{3}I_{P2}}{\sqrt{3}I_{P1}} = \frac{N_1}{N_2}$$



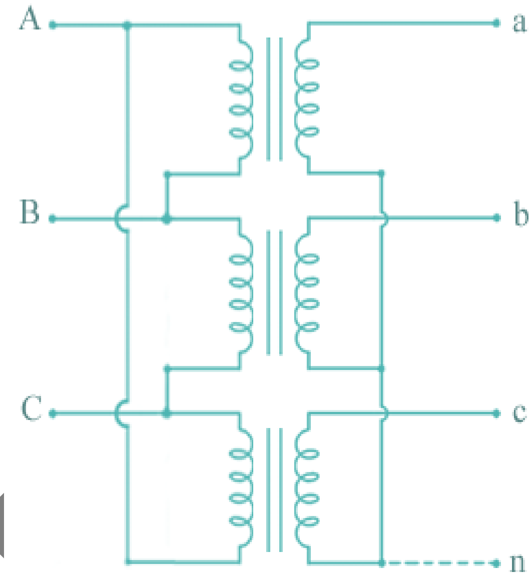
(3) Y-Δ (Star-Delta) connection: It is used where the voltage is to be stepped down for example at the end of a transmission line.

$$\text{Voltage ratio} = \frac{V_{L2}}{V_{L1}} = \frac{V_{P2}}{\sqrt{3}V_{P1}} = \frac{1}{\sqrt{3}}\frac{N_2}{N_1}$$

$$\text{Current ratio} = \frac{I_{L2}}{I_{L1}} = \frac{\sqrt{3}I_{P2}}{I_{P1}} = \sqrt{3}\frac{N_1}{N_2}$$



(4) Δ-Y (Delta- Star) connection: It is used where the voltage is to be stepped up for example at the beginning of the transmission line. This arrangement is very popular in distribution system because it can be used to serve both the three phase equipment and the single phase lighting load.



$$\text{Voltage ratio} = \frac{V_{L2}}{V_{L1}} = \frac{\sqrt{3}V_{P2}}{V_{P1}} = \sqrt{3}\frac{N_2}{N_1}$$

$$\text{Current ratio} = \frac{I_{L2}}{I_{L1}} = \frac{I_{P2}}{\sqrt{3}I_{P1}} = \frac{1}{\sqrt{3}}\frac{N_1}{N_2}$$

4.18 FORMULA USE TO SOLVE NUMERICAL OF THE TRANSFORMER

Equivalent resistance of the transformer referred to primary (R_{01})

$$R_{01} = R_1 + \frac{R_2}{k^2}$$

Equivalent reactance of the transformer referred t`o primary (X_{01})

$$X_{01} = X_1 + \frac{X_2}{k^2}$$

Equivalent impedance of the transformer referred to primary (Z_{01})

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

Equivalent resistance of the transformer referred to secondary (R_{02})

$$R_{02} = k^2 R_1 + R_2$$

Equivalent reactance of the transformer referred to secondary (X_{02})

$$X_{02} = k^2 X_1 + X_2$$

Equivalent impedance of the transformer referred to secondary (Z_{02})

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$

Total copper loss $P_c = I_2^2 R_{02}$

Efficiency

$$\eta\% = \frac{x\, V_2 I_2 \cos \emptyset_2}{x\, V_2 I_2 \cos \emptyset_2 + P_i + x^2 P_c} \times 100$$

Where

$V_2 I_2$ = VA rating of the transformer (full load)

x is the fraction of full load

$$\text{Load corresponding to the mmaximum efficiency} = \text{full load} \times \sqrt{\frac{P_i}{P_c}}$$

How to find x

$$x = \frac{\text{Load at which we have to find the efficiency}}{\text{full load}}$$

Suppose we have to find the efficiency at

(1) Full load: x=1
(2) Half load: x=0.5
(3) 75% of full load: x=0.75

Example 8.1 A 30 kVA 2000/200 V, single phase 50 Hz transformer has a primary resistance of 2.5 Ω and reactance 3.5 Ω. The secondary resistance and reactance are 0.012 Ω and 0.01 Ω respectively. Determine

(1) Equivalent resistance referred to primary
(2) Equivalent reactance referred to primary
(3) Equivalent impedance referred to primary
(4) Equivalent resistance referred to secondary
(5) Equivalent reactance referred to secondary
(6) Equivalent impedance referred to secondary
(7) Total copper loss

Solution: Given $V_2 I_2 = 30 \times 10^3, \frac{V_1}{V_2} = \frac{2000}{200}, R_1 = 2.5\ \Omega, X_1 = 3.5\ \Omega, R_2 = 0.012\ \Omega, X_2 = 0.01\ \Omega$

$$K = \frac{V_2}{V_1} = \frac{200}{2000} = 0.1$$

(1) Equivalent resistance of the transformer referred to primary (R_{01})

$$R_{01} = R_1 + \frac{R_2}{k^2} = 2.5 + \frac{0.012}{(0.1)^2} = 2.5 + 1.2 = 3.7\ \Omega\ \textbf{Ans.}$$

(2) Equivalent reactance of the transformer referred to primary (X_{01})

$$X_{01} = X_1 + \frac{X_2}{k^2} = 3.5 + \frac{0.01}{(0.1)^2} = 3.5 + 1 = 4.5\ \Omega\ \textbf{Ans.}$$

(3) Equivalent impedance of the transformer referred to primary (Z_{01})

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{(3.7)^2 + (4.5)^2} = 5.826\ \Omega\ \textbf{Ans.}$$

(4) Equivalent resistance of the transformer referred to secondary (R_{02})

$$R_{02} = k^2 R_1 + R_2 = (0.1)^2 2.5 + 0.012 = 0.025 + 0.012 = 0.037\ \Omega\ \textbf{Ans.}$$

(5) Equivalent reactance of the transformer referred to secondary (X_{02})

$$X_{02} = k^2X_1 + X_2 = (0.1)^2 3.5 + 0.01 = 0.035 + 0.01 = 0.045\ \Omega\ \textbf{Ans}.$$

(6) Equivalent impedance of the transformer referred to secondary (Z_{02})

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{(0.037)^2 + (0.045)^2} = 0.0582\ \Omega\ \textbf{Ans}.$$

(7) Total copper loss

$$P_c = I_2^2 R_{02}$$

$$I_2 = \frac{30 \times 10^3}{V_2} = \frac{30 \times 10^3}{200} = 150\ \text{A}$$

$$P_c = I_2^2 R_{02} = 150^2 \times 0.037 = 832.5\ \text{W}\ \textbf{Ans}.$$

Example 8.2 A 25 kVA single phase transformer the iron and copper losses are 400 W and 500 W respectively. Calculate the

(1) Efficiency at full load and 0.8 p.f.
(2) Efficiency at half load and 0.8 p.f.
(3) Efficiency at 70% of full load and 0.6 p.f.
(4) Efficiency at 18 kVA and 0.6 p.f.
(5) Load corresponding the maximum efficiency
(6) Maximum efficiency at 0.8 p.f.

Solution: Given $V_2I_2 = 25\text{kVA} = 25 \times 10^3\ \text{VA}, P_i = 400\ \text{W}, P_c = 500\ \text{W}$

(1) Efficiency at full load and 0.8 p.f.

$$x = 1, \cos\emptyset_2 = 0.8$$

$$\eta\% = \frac{x\, V_2I_2 \cos\emptyset_2}{x\, V_2I_2 \cos\emptyset_2 + P_i + x^2P_c} \times 100 = \frac{1 \times 25 \times 10^3 \times 0.8}{(1 \times 25 \times 10^3 \times 0.8) + 400 + (1)^2 \times 500} \times 100$$

$$= \frac{20000}{20900} \times 100 = 95.69\%\ \textbf{Ans}.$$

(2) Efficiency at half load and 0.8 p.f.

$$x = 0.5, \cos\emptyset_2 = 0.8$$

$$\eta\% = \frac{x\, V_2I_2 \cos\emptyset_2}{x\, V_2I_2 \cos\emptyset_2 + P_i + x^2P_c} \times = \frac{0.5 \times 25 \times 10^3 \times 0.8}{(0.5 \times 25 \times 10^3 \times 0.8) + 400 + (0.5)^2 \times 500} \times 100$$

$$= \frac{10000}{10525} \times 100 = 95.01\%\ \textbf{Ans}.$$

(3) Efficiency at 70% of full load and 0.6 p.f.

$$x = 0.7, \cos\emptyset_2 = 0.6$$

$$\eta\% = \frac{x\,V_2I_2\cos\emptyset_2}{x\,V_2I_2\cos\emptyset_2 + P_i + x^2P_c} \times 100 = \frac{0.7 \times 25 \times 10^3 \times 0.6}{(0.7 \times 25 \times 10^3 \times 0.6) + 400 + (0.7)^2 \times 500} \times 100$$
$$= \frac{10500}{11145} \times 100 = 94.21\%\ \textbf{Ans}.$$

(4) Efficiency at 18 kVA and 0.6 p.f.

$$x = \frac{18}{25} = 0.72, \cos\emptyset_2 = 0.6$$

$$\eta\% = \frac{x\,V_2I_2\cos\emptyset_2}{x\,V_2I_2\cos\emptyset_2 + P_i + x^2P_c} \times 100 = \frac{0.72 \times 25 \times 10^3 \times 0.6}{(0.72 \times 25 \times 10^3 \times 0.6) + 400 + (0.72)^2 \times 500} \times 100$$
$$= \frac{10800}{11459.2} \times 100 = 94.25\%\ \textbf{Ans}.$$

(5) Load corresponding the maximum efficiency

$$\text{Load corresponding to the mmaximum efficiency} = \text{full load} \times \sqrt{\frac{P_i}{P_c}} = 25 \times \sqrt{\frac{400}{500}}$$
$$= 22.36\text{ kVA}\ \textbf{Ans}.$$

(6) Maximum efficiency at 0.8 p.f.

$$x = \frac{22.36}{25} = 0.8944, \cos\emptyset_2 = 0.8$$

$$\eta\% = \frac{x\,V_2I_2\cos\emptyset_2}{x\,V_2I_2\cos\emptyset_2 + P_i + x^2P_c} \times 100 = \frac{0.8944 \times 25 \times 10^3 \times 0.8}{(0.8944 \times 25 \times 10^3 \times 0.8) + 400 + (0.8944)^2 \times 500} \times 100$$
$$= \frac{17888}{18688} \times 100 = 95.72\%\ \textbf{Ans}.$$

Example 8.3 A 400 kVA single phase transformer the iron and copper losses are 900 W and 1200 W respectively. Calculate the efficiency at half load and 0.8 p.f.. Also find the maximum efficiency.

Solution: Given $V_2I_2 = 400\text{ kVA} = 400 \times 10^3\text{ VA}, P_i = 900\text{ W}, P_c = 1200$

Efficiency at half load and 0.8 p.f.

$$x = 0.5, \cos\emptyset_2 = 0.8$$

$$\eta\% = \frac{x\,V_2I_2\cos\emptyset_2}{x\,V_2I_2\cos\emptyset_2 + P_i + x^2P_c} \times 100 = \frac{0.5 \times 400 \times 10^3 \times 0.8}{(0.5 \times 400 \times 10^3 \times 0.8) + 900 + (0.5)^2 \times 1200} \times 100$$
$$= \frac{160000}{161200} \times 100 = 99.256\%\ \textbf{Ans}.$$

Maximum efficiency at 0.8 p.f

Load corresponding the maximum efficiency

$$\text{Load corresponding to the mmaximum efficiency} = \text{full load } \times \sqrt{\frac{P_i}{P_c}} = 400 \times \sqrt{\frac{900}{1200}} = 346.41 \text{ kVA}$$

$$x = \frac{346.41}{400} = 0.866, \cos\emptyset_2 = 0.8$$

$$\eta\% = \frac{x\, V_2 I_2 \cos\emptyset_2}{x\, V_2 I_2 \cos\emptyset_2 + P_i + x^2 P_c} \times 100 = \frac{0.866 \times 400 \times 10^3 \times 0.8}{(0.866 \times 400 \times 10^3 \times 0.8) + 900 + (0.866)^2 \times 1200} \times 100$$

$$= \frac{277120}{278920} \times 100 = 99.35\% \ \mathbf{Ans}.$$

Example 8.4 A 200 kVA single phase transformer the iron and copper losses are 600 W and 800 W respectively. Calculate the efficiency at 75% of full load at unity p.f.. Also find the maximum efficiency.

Solution: Given $V_2 I_2 = 200 \text{ kVA} = 200 \times 10^3 \text{ VA}, P_i = 600 \text{ W}, P_c = 800 \text{ W}$

Efficiency at 75% of full load at unity p.f.

$$x = 0.75, \cos\emptyset_2 = 1$$

$$\eta\% = \frac{x\, V_2 I_2 \cos\emptyset_2}{x\, V_2 I_2 \cos\emptyset_2 + P_i + x^2 P_c} \times 100 = \frac{0.75 \times 200 \times 10^3 \times 1}{(0.75 \times 200 \times 10^3 \times 1) + 600 + (0.75)^2 \times 800} \times 100$$

$$= \frac{150000}{151050} \times 100 = 99.30\% \ \mathbf{Ans}.$$

Maximum efficiency at unity p.f

Load corresponding the maximum efficiency

$$\text{Load corresponding to the maximum efficiency} = \text{full load } \times \sqrt{\frac{P_i}{P_c}} = 200 \times \sqrt{\frac{600}{800}} = 173.20 \text{ kVA}$$

$$x = \frac{173.20}{200} = 0.866, \cos\emptyset_2 = 1$$

$$\eta\% = \frac{x\, V_2 I_2 \cos\emptyset_2}{x\, V_2 I_2 \cos\emptyset_2 + P_i + x^2 P_c} \times 100 = \frac{0.866 \times 200 \times 10^3 \times 1}{(0.866 \times 200 \times 10^3 \times 1) + 600 + (0.866)^2 \times 800} \times 100$$

$$= \frac{173200}{174000} \times 100 = 99.31\% \ \mathbf{Ans}.$$

Example 8.5 The efficiency of a 500 MVA, single phase transformer is 98.70% at full load 0.9 power factor and 99.10% at half load and unity power factor. Find,

(i) Iron loss at full load and half load
(ii) Copper loss at full load and half load

Solution: Given $V_2 I_2 = 500 \text{ MVA} = 500 \times 10^6$

Case 1 $\eta = 98.70\%, x = 1, \text{p.f.} = 0.9$

$$\eta\% = \frac{x\, V_2 I_2 \cos\phi_2}{x\, V_2 I_2 \cos\phi_2 + P_i + x^2 P_c} \times 100$$
$$98.70\% = \frac{1 \times 500 \times 10^6 \times 0.9}{(1 \times 500 \times 10^6 \times 0.9) + P_i + (1)^2 \times P_c} \times 100$$
$$P_i + P_c = 5.93\ \text{MW}$$

Case 2 $\eta = 99.10\%, x = 0.5, \text{p.f.} = 1$

$$\eta\% = \frac{x\, V_2 I_2 \cos\phi_2}{x\, V_2 I_2 \cos\phi_2 + P_i + x^2 P_c} \times 100$$
$$99.10\% = \frac{0.5 \times 500 \times 10^6 \times 1}{(0.5 \times 500 \times 10^6 \times 1) + P_i + (0.5)^2 \times P_c} \times 100$$
$$P_i + 0.25 P_c = 2.27\ \text{MW}$$
$$P_i = 1.05\ \text{MW} \qquad P_c = 4.88\ \text{MW}$$

(i) Iron loss at full load = 1.05 MW **Ans.** Iron loss at half load = 1.05 MW **Ans.**

(ii) Copper loss at full load = 4.88 MW **Ans.** Copper loss at half load = 1.22 MW **Ans.**

Example 8.6 The maximum efficiency of a 150 kVA, single phase transformer is 97.12%, that obtained at 80% of full load at 0.9 p.f. lagging. Find the efficiency at 70% of full load at 0.7 p.f.

Solution: Given $V_2 I_2 = 150\ \text{kVA} = 150 \times 10^3, \eta_{max} = 97.12\%, x = 0.8, \text{p.f.} = 0.9$

$$\eta_{max}\% = \frac{x\, V_2 I_2 \cos\phi_2}{x\, V_2 I_2 \cos\phi_2 + P_i + x^2 P_c} \times 100$$
$$97.12\% = \frac{0.8 \times 150 \times 10^3 \times 0.9}{(0.8 \times 150 \times 10^3 \times 0.9) + P_i + (0.8)^2 \times P_c} \times 100$$
$$P_i + 0.64 P_c = 3202.64$$

This efficiency will be maximum when iron loss and copper loss are equal

$$P_i = 0.64 P_c$$

So $$P_i + P_i = 3202.64 \Rightarrow 2P_i = 3202.64 \Rightarrow P_i = 1601.32\ \text{W}$$

$$0.64 P_c = 1601.32 \Rightarrow P_c = 2502.06\ \text{W}$$

Efficiency at 70% of full load at 0.7 p.f.

$$\text{Given } x = 0.7, \text{p.f.} = 0.7$$

$$\eta\% = \frac{x\, V_2 I_2 \cos\phi_2}{x\, V_2 I_2 \cos\phi_2 + P_i + x^2 P_c} \times 100$$
$$= \frac{0.7 \times 150 \times 10^3 \times 0.7}{(0.7 \times 150 \times 10^3 \times 0.7) + 1601.32 + (0.7)^2 \times 2502.06} \times 100 = \frac{73500}{76327.33} \times 100$$
$$= 96.30\%\ \textbf{Ans.}$$