UNIT 2

THREE PHASE AC CIRCUITS

5.1 INTRODUCTION

Three phase AC circuits mean the circuit which consists of three phase (winding) which is supply by three AC voltages.

Three phase AC supply: The supply which consist three AC voltages those are having phase difference from each other is called the three phase AC supply. In balanced condition they are having a phase difference 120° from each other. The three phase supply can be represents by the eq. (i), (ii) and (ii)

$$v_1(\text{R phase}) = V_{m1} \sin \omega t \qquad \text{.........(i)}$$

$$v_2(\text{Y phase}) = V_{m2} \sin(\omega t - 120°) \qquad \text{.........(ii)}$$

$$v_3(\text{B phase}) = V_{m3} (\sin \omega t - 240°) \qquad \text{.........(iii)}$$

Where V_{m1}, V_{m2}, V_{m3} are the maximum values of $v_1, v_2,$ and v_3 respectively.

Naming of the phases: The three phases are denoted by R Phase, Y Phase and B Phase.

Phase sequence: The sequence in which these three phase attained there positive maximum value is called the phase sequence. The phase sequence is RYB.

Double subscript notation: The voltage between R and Y is denoted by V_{RY}. This type of notation is called the double subscript notation. $V_{RY} = -V_{YR}$

Balanced supply: A three phase supply is called balanced if the three voltages are equal in magnitude and having phase difference 120° from each other.

Unbalanced supply: A three phase supply is called unbalanced if the three voltages are either unequal in magnitude or not having a phase difference 120° from each other.

Balanced Load: A three phase load is said to be balanced when all the impedance are equal in magnitude and the phase angle of all of them are equal and of the same nature.

Unbalanced Load: A three phase load is said to be unbalanced when either all the impedance are unequal in magnitude or the phase angle of all of them are unequal.

Line Voltage: The voltage between two line wires is called the line voltage. Example V_{RY}, V_{YB}, V_{BR}

Phase Voltage: The voltage between line wire and neutral wire is called the phase voltage. Example V_{RN}, V_{YN}, V_{BN}

Line Current: The current flowing in the line wire is called the Line current. Example I_R, I_Y, I_B

Phase Current: The current flowing in the phase wire is called the phase current. Example I_{RY}, I_{YB}, I_{BR}

5.2 ADVANTAGE OF THREE PHASE SYSTEM OVER SINGLE PHASE SYSTEM

The advantage of three phase system over single phase system of the same rating is

(1) The output of three phase machine is higher.

(2) The efficiency of three phase machine is higher.
(3) The power factor of three phase machine is higher.
(4) The reliability of three phase machine is higher.
(5) The size of three phase machine is smaller.
(6) The cost of three phase machine is smaller.

5.3 INTERCONNECTION OF THREE PHASES

The two method of connecting the three phase are star connection and delta connection.

5.4 STAR CONNECTION

If the second terminal of three phases is joined together as shown in fig. 5.1 (a), that connection is called star connection.

Relation between line voltage and phase voltage: The star connection showing all the line voltages and phase voltages are shown in fig. 5.1 (a)

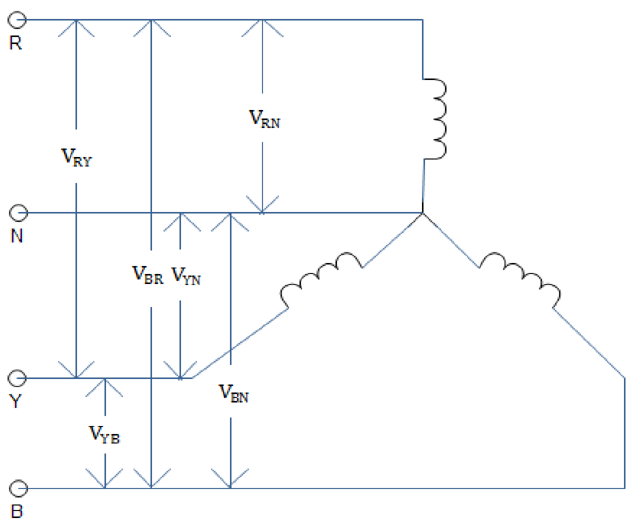

Fig. 5.1 (a)

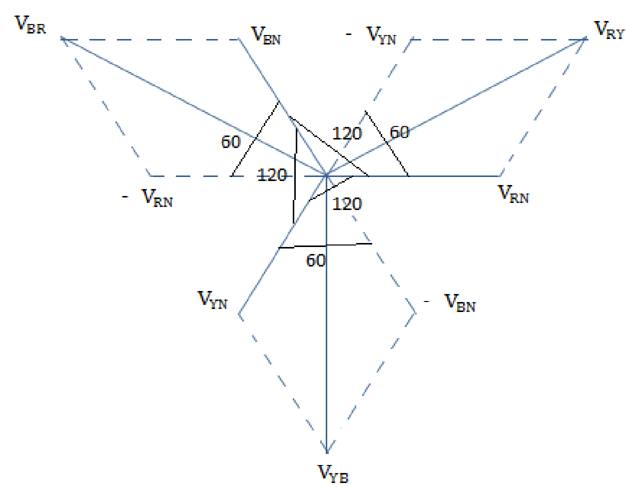

Fig 5.1 (b)

From the connection diagram

$$V_{RY} = V_{RN} - V_{YN} = (V_{RN}) + (-V_{YN})$$
$$V_{YB} = V_{YN} - V_{BN} = (V_{YN}) + (-V_{BN})$$
$$V_{BR} = V_{BN} - V_{RN} = (V_{BN}) + (-V_{RN})$$

The line voltage V_{RY} is the phasor sum of (V_{RN}) and $(-V_{YN})$ shown in phasor diagram 5.1 (b)
Similarly line voltage V_{YB} is the phasor sum of (V_{YN}) and $(-V_{BN})$ shown in phasor diagram 5.1 (b)
Similarly line voltage V_{BR} is the phasor sum of (V_{BN}) and $(-V_{RN})$ shown in phasor diagram 5.1 (b)

From the phasor diagram shown in fig. 5.1 (b)

$$V_{RY} = \sqrt{V_{RN}^2 + V_{YN}^2 + 2V_{RN}V_{YN}\cos 60°}$$

It the supply is balanced

$$V_{RN} = V_{YN} = V_{BN} = V_{Ph}(\text{Phase voltage})$$

And

$$V_{RY} = V_{YB} = V_{BR} = V_L(\text{Line voltage})$$

Then

$$V_L = \sqrt{V_{Ph}^2 + V_{Ph}^2 + 2V_{Ph}V_{Ph}\cos 60°} = \sqrt{V_{Ph}^2 + V_{Ph}^2 + 2V_{Ph}^2\frac{1}{2}} = \sqrt{3V_{Ph}^2} = \sqrt{3}V_{Ph}$$

So the relation between the line voltage and phase voltage

$$V_L = \sqrt{3}V_{Ph}$$

Relation between line current and phase current: In star connection shown in fig. 5.1 (a) line wire and phase wire are connected in series so in star connection

$$I_L = I_{Ph}$$

Power in star connection:

$$\text{power in 3 phase circuit} = 3 \times \text{power in 1 phase circuit}$$

$$P = 3V_{Ph}I_{Ph}\cos\emptyset$$

We know in star connection $V_L = \sqrt{3}V_{Ph}$ and $I_L = I_{Ph}$ so

$$P = 3\frac{V_L}{\sqrt{3}}I_L\cos\emptyset$$

$$\text{Active Power (P)} = \sqrt{3}V_LI_L\cos\emptyset \qquad \text{W}$$

Similarly

$$\text{Reactive Power (Q)} = \sqrt{3}V_LI_L\sin\emptyset \qquad \text{VAR}$$

$$\text{Apparent Power (S)} = \sqrt{3}V_LI_L \qquad \text{VA}$$

5.5 DELTA CONNECTION

If the second terminal of the one phase is connected to first terminal of the second phase and forming a closed path as shown in fig. 5.2 (a), that connection is called delta connection.

Relation between line voltage and phase voltage: In delta connection shown in fig. 5.2 (a) neutral wire is absent so on delta connection

$$V_L = V_{Ph}$$

Relation between line current and phase current: The delta connection showing all the line currents and phase currents are shown in fig. 5.2 (a)

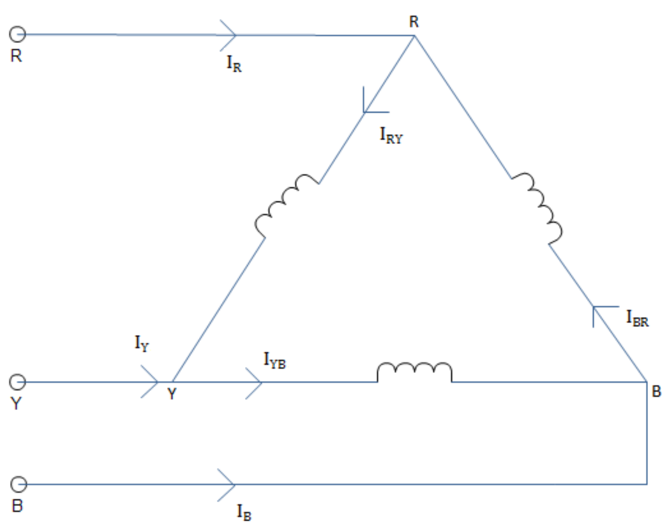

Fig. 5.2 (a)

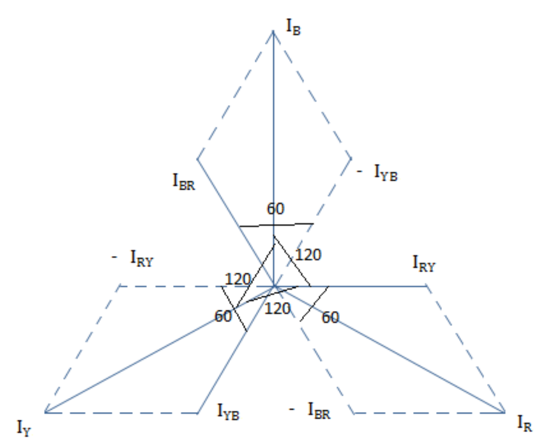

Fig. 5.2 (b)

From the connection diagram shown in fig. 5.2 (a)

$$I_R = I_{RY} - I_{BR} = (I_{RY}) + (-I_{BR})$$
$$I_Y = I_{YB} - I_{RY} = (I_{YB}) + (-I_{RY})$$
$$I_B = I_{BR} - I_{YB} = (I_{BR}) + (-I_{YB})$$

The line current I_R is the phasor sum of (I_{RY}) and $(-I_{BR})$ shown in phasor diagram 5.2 (b)
Similarly line current I_Y is the phasor sum of (I_{YB}) and $(-I_{RY})$ shown in phasor diagram 5.2 (b)
Similarly line current I_B is the phasor sum of (I_{BR}) and $(-I_{YB})$ shown in phasor diagram 5.2 (b)

From the phasor diagram shown in fig. 5.2 (b)

$$I_R = \sqrt{I_{RY}^2 + I_{BR}^2 + 2I_{RY}I_{BR}\cos 60°}$$

It the supply is balanced

$$I_{RY} = I_{YB} = I_{BR} = I_{Ph}(\text{Phase voltage})$$

And

$$I_R = I_Y = I_B = I_L(\text{Line voltage})$$

Then

$$I_L = \sqrt{I_{Ph}^2 + I_{Ph}^2 + 2I_{Ph}I_{Ph}\cos 60°} = \sqrt{I_{Ph}^2 + I_{Ph}^2 + 2I_{Ph}^2\frac{1}{2}} = \sqrt{3I_{Ph}^2} = \sqrt{3}I_{Ph}$$

So the relation between the line current and phase current

$$I_L = \sqrt{3}I_{Ph}$$

Power in delta connection:

$$\text{power in 3 phase circuit} = 3 \times \text{power in 1 phase circuit}$$
$$P = 3V_{Ph}I_{Ph}\cos\emptyset$$

We know in star connection $V_L = V_{Ph}$ and $I_L = \sqrt{3}I_{Ph}$ so

$$P = 3V_L\frac{I_L}{\sqrt{3}}\cos\emptyset$$
$$\text{Active Power (P)} = \sqrt{3}V_LI_L\cos\emptyset \quad \text{W}$$

Similarly

$$\text{Reactive Power (Q)} = \sqrt{3}V_LI_L\sin\emptyset \quad \text{VAR}$$
$$\text{Apparent Power (S)} = \sqrt{3}V_LI_L \quad \text{VA}$$

5.6 FORMULA USE TO SOLVE NUMERICAL FOR STAR DELTA

The supply voltage and supply current means Line voltage and Line current respectively.
Never try to calculate Line current directly. First find the phase current using relation (4) then find the line current using relation (2).

S. No.	RELATION	STAR CONNECTION	DELTA CONNECTION
1	LINE VOLTAGE AND PHASE VOLTAGE	$V_L = \sqrt{3}V_{Ph}$	$V_L = V_{Ph}$
2	LINE CURRENT AND PHASE CURRENT	$I_L = I_{Ph}$	$I_L = \sqrt{3}I_{Ph}$

3	POWER FACTOR	$\cos\emptyset = \frac{R_{Ph}}{Z_{Ph}}$
4	PHASE VOLTAGE, PHASE CURRENT, PHASE IMPEDANCE	$V_{Ph} = I_{Ph}Z_{Ph}$
5	Active Power (P)	$(P) = \sqrt{3}V_L I_L \cos\emptyset \ W$
6	Reactive Power (Q)	$(Q) = \sqrt{3}V_L I_L \sin\emptyset \ \ VAR$
7	Apparent Power (S)	$(S) = \sqrt{3}V_L I_L \ VA$

Example 5.1 Three similar coil each having resistance 8 Ω and inductance 0.8 H are connected in star across 400 V, 50 Hz 3 phase supply. Find (1) Line current (2) Power factor (3) Active power (4) Reactive power (5) Apparent power

Solution: Given $R_{Ph} = 8\ \Omega, L_{Ph} = 0.8\ H, V_L = 400\ V, f = 50\ Hz,$ Connection is STAR

$$X_{LPh} = 2\pi f L_{Ph} = 2 \times \pi \times 50 \times 0.8 = 251.3\ \Omega$$

$$Z_{Ph} = \sqrt{R_{Ph}^2 + X_{LPh}^2} = \sqrt{8^2 + 251.3^2} = 251.4\ \Omega$$

For star connection

$$V_L = \sqrt{3}V_{Ph} \Rightarrow V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.9\ V$$

We know

$$V_{Ph} = I_{Ph}Z_{Ph} \Rightarrow I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{230.9}{251.4} = 0.92\ A$$

(1) Line current: for star connection

$$I_L = I_{Ph} = 0.92\ A\ \textbf{Ans.}$$

(2) Power factor: We know

$$\cos\emptyset = \frac{R_{Ph}}{Z_{Ph}} = \frac{8}{251.4} = 0.032\ \textbf{Ans.}$$

(3) Active power: We know

$$P = \sqrt{3}V_L I_L \cos\emptyset = \sqrt{3} \times 400 \times 0.92 \times 0.032\ = 20.4\ W\ \textbf{Ans.}$$

(4) Reactive power: We know

$$Q = \sqrt{3}V_L I_L \sin\emptyset = \sqrt{3} \times 400 \times 0.92 \times 0.999\ = 636.75\ VAR\ \textbf{Ans.}`$$

(5) Apparent power: We know

$$S = \sqrt{3}V_L I_L = \sqrt{3} \times 400 \times 0.92\ = 637.4\ VA\ \textbf{Ans.}$$

Example 5.2 Three phase 440 V, 50 Hz supply is connected to 3 phase delta connected load with each phase having a resistance 20 Ω, inductance of 100 mH and capacitance of 120 µF in series. Find (1) Line current (2) Power factor (3) Active power (4) Reactive power (5) Apparent power

Solution: Given

$R_{Ph} = 20\ \Omega,\ L_{Ph} = 100\ mH,\ C_{Ph} = 120\ \mu F,\ V_L = 440\ V,\ f = 50\ Hz,$ Connection is DELTA

$$X_{LPh} = 2\pi f L_{Ph} = 2 \times \pi \times 50 \times 100 \times 10^{-3} = 31.4\ \Omega$$

$$X_{CPh} = \frac{1}{2\pi f C_{Ph}} = \frac{1}{2 \times \pi \times 50 \times 120 \times 10^{-6}} = 26.53\ \Omega$$

$$Z_{Ph} = \sqrt{R_{Ph}^2 + (X_{LPh} - X_{CPh})^2} = \sqrt{20^2 + (31.4 - 26.53)^2} = 20.58\ \Omega$$

For delta connection

$$V_L = V_{Ph} = 440\ V$$

We know

$$V_{Ph} = I_{Ph} Z_{Ph} \Rightarrow I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{440}{20.58} = 21.38\ A$$

(1) Line current: for delta connection

$$I_L = \sqrt{3} I_{Ph} = \sqrt{3} \times 21.38 = 37.03\ A\ \textbf{Ans.}$$

(2) Power factor: We know

$$\cos\emptyset = \frac{R_{Ph}}{Z_{Ph}} = \frac{20}{20.58} = 0.972\ \textbf{Ans.}$$

(3) Active power: We know

$$P = \sqrt{3} V_L I_L \cos\emptyset = \sqrt{3} \times 440 \times 37.03 \times 0.972 = 27430.4\ W = 27.43\ kW\ \textbf{Ans.}$$

(4) Reactive power: We know

$$Q = \sqrt{3} V_L I_L \sin\emptyset = \sqrt{3} \times 440 \times 37.03 \times 0.235 = 6631.8\ VAR = 6.63\ kVAR\ \textbf{Ans.}`$$

(5) Apparent power: We know

$$S = \sqrt{3} V_L I_L = \sqrt{3} \times 440 \times 37.03 = 28220\ VA = 28.22\ kVA\ \textbf{Ans.}$$

Example 5.3 A star connected load has impedance 2+j3 in each phase and is connected across a balanced 440 V, 50 Hz 3 phase supply. Obtain (1) Line current (2) Power factor (3) Active power (4) Reactive power (5) Apparent power

Solution: Given $Z_{Ph} = 2 + j3, V_L = 440\ V,\ f = 50\ Hz,$ Connection is STAR

$$Z_{Ph} = 2 + j3 = 3.6\angle 56.3°$$

For star connection

$$V_L = \sqrt{3} V_{Ph} \Rightarrow V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254\ V$$

We know

$$V_{Ph} = I_{Ph} Z_{Ph} \Rightarrow I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{254}{3.6\angle 56.3°} = 70.56\angle -56.3°A$$

(1) Line current: for star connection

$$I_L = I_{Ph} = 70.56\angle -56.3°A\ \textbf{Ans.}$$

(2) Power factor: We know

$$\cos\emptyset = \cos(-56.3°) = 0.555\ \textbf{Ans.}$$

(3) Active power: We know

$$P = \sqrt{3} V_L I_L \cos\emptyset = \sqrt{3} \times 440 \times 70.56 \times 0.555 = 29844.5\ W = 29.84\ kW\ \textbf{Ans.}$$

(4) Reactive power: We know

$$Q = \sqrt{3} V_L I_L \sin\emptyset = \sqrt{3} \times 440 \times 70.56 \times (-0.832) = -44739\ VAR$$
$$= -44.74\ kVAR\ \textbf{Ans.}`$$

(5) Apparent power: We know

$$S = \sqrt{3}V_L I_L = \sqrt{3} \times 440 \times 70.56 = 53773.9\ \text{VA} = 53.77\ \text{kVA}\ \textbf{Ans}.$$

Example 5.4 A delta connected load has impedance 5+j8 in each phase and is connected across a balanced 400 V, 50 Hz 3 phase supply. Obtain (1) Line current (2) Power factor (3) Active power (4) Reactive power (5) Apparent power

Solution: Given $Z_{Ph} = 5 + j8, V_L = 400\ \text{V},\ f = 50\ \text{Hz}$, Connection is DELTA

$$Z_{Ph} = 5 + j8 = 9.43\angle 57.99$$

For delta connection

$$V_L = V_{Ph} = 400\ \text{V}$$

We know

$$V_{Ph} = I_{Ph} Z_{Ph} \Rightarrow I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{400}{9.43\angle 57.99} = 42.41\angle -57.99\ \text{A}$$

(1) Line current: for delta connection

$$I_L = \sqrt{3} I_{Ph} = \sqrt{3} \times 42.41 = 73.46\ \text{A}\ \textbf{Ans}.$$

(2) Power factor: We know

$$\cos\emptyset = \cos(-57.99) = 0.53\ \textbf{Ans}.$$

(3) Active power: We know

$$P = \sqrt{3}V_L I_L \cos\emptyset = \sqrt{3} \times 400 \times 73.46 \times 0.53 = 26974\ \text{W} = 26.97\ \text{kW}\ \textbf{Ans}.$$

(4) Reactive power: We know

$$Q = \sqrt{3}V_L I_L \sin\emptyset = \sqrt{3} \times 400 \times 73.46 \times (-0.848) = 43158\ \text{VAR} = 43.15\ \text{kVAR}\ \textbf{Ans}.`$$

(5) Apparent power: We know

$$S = \sqrt{3}V_L I_L = \sqrt{3} \times 400 \times 73.46 = 50894.6\ \text{VA} = 50.89\ \text{kVA}\ \textbf{Ans}.$$

Example 5.5 A balanced 3 phase star connected load of 150 kW takes a lagging current of 100 Amp From a 1000 V, 50 Hz supply. Find the circuit constant of the load per phase.

Solution: Given $P = 150\ \text{kW}, I_L = 100\ \text{A(lagging)}, V_L = 1000\ \text{V}, f = 50\ \text{Hz}$, Connection is STAR

The current is lagging so R-L connected in series in each phase of star, so circuit constant is R_{Ph}, L_{Ph}

For star connection

$$V_L = \sqrt{3}V_{Ph} \Rightarrow V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{1000}{\sqrt{3}} = 577.35\ \text{V}$$

$$I_L = I_{Ph} = 100\ \text{A}$$

We know

$$V_{Ph} = I_{Ph} Z_{Ph} \Rightarrow Z_{Ph} = \frac{V_{Ph}}{I_{Ph}} = \frac{577.35}{100} = 5.77\ \Omega$$

$$P = \sqrt{3}V_L I_L \cos\emptyset \ \Rightarrow \cos\emptyset = \frac{P}{\sqrt{3}V_L I_L} = \frac{150 \times 10^3}{\sqrt{3} \times 1000 \times 100} = 0.866$$

R_{Ph}: We know

$$\cos\emptyset = \frac{R_{Ph}}{Z_{Ph}} \Rightarrow R_{Ph} = Z_{Ph}\cos\emptyset = 5.77 \times 0.866 = 5\,\Omega\ \textbf{Ans}.$$

L_{Ph}: We know

$$Z_{Ph} = \sqrt{R_{Ph}^2 + X_{LPh}^2} \Rightarrow X_{LPh} = \sqrt{(Z_{Ph})^2 - (R_{Ph})^2} = \sqrt{5.77^2 - 5^2} = 2.88\,\Omega$$

$$X_{LPh} = 2\pi f L_{Ph} \Rightarrow L_{Ph} = \frac{X_{LPh}}{2\pi f} = \frac{2.88}{2 \times \pi \times 50} = 9.16 \times 10^{-3} = 9.16\text{ mH } \textbf{Ans}.$$

Example 5.6 A balanced 3 phase delta connected load connected across 440V,50 Hz, 3 Ø AC supply, takes a line current of 20 A at power factor of 0.8 leading. Find (1) Phase current (2) Phase Impedance (3) Find the circuit constant of the load per phase (4) Power drawn by each phase

Solution: Given $\cos\emptyset = 0.8(\text{leading}), I_L = 20\text{ A}, V_L = 440\text{ V}, f = 50\text{ Hz}$, Connection is DELTA

The p.f. is leading so R-C connected in series in each phase of delta, so circuit constant is R_{Ph}, C_{Ph}

For delta connection

$$V_L = V_{Ph} = 440$$

(1) Phase current (I_{Ph}): For delta connection

$$I_L = \sqrt{3}I_{Ph} \Rightarrow I_{Ph} = \frac{I_L}{\sqrt{3}} = \frac{20}{\sqrt{3}} = 11.54\text{ A } \textbf{Ans}.$$

(2) Phase impedance (Z_{Ph}): We know

$$V_{Ph} = I_{Ph}Z_{Ph} \Rightarrow Z_{Ph} = \frac{V_{Ph}}{I_{Ph}} = \frac{440}{11.54} = 38.13\,\Omega\ \textbf{Ans}.$$

(3) Circuit constant (R_{Ph}, C_{Ph}):

R_{Ph}: We know

$$\cos\emptyset = \frac{R_{Ph}}{Z_{Ph}} \Rightarrow R_{Ph} = Z_{Ph}\cos\emptyset = 38.13 \times 0.8 = 30.5\,\Omega\ \textbf{Ans}.$$

C_{Ph}:we know

$$Z_{Ph} = \sqrt{R_{Ph}^2 + X_{CPh}^2} \Rightarrow X_{CPh} = \sqrt{(Z_{Ph})^2 - (R_{Ph})^2} = \sqrt{38.13^2 - 30.5^2} = 22.88\,\Omega$$

$$X_{CPh} = \frac{1}{2\pi f C_{Ph}} \Rightarrow C_{Ph} = \frac{1}{2\pi f X_{CPh}} = \frac{1}{2 \times \pi \times 50 \times 22.88} = 1.39 \times 10^{-4} = 139\,\mu\text{F } \textbf{Ans}.$$

(4) Power drawn by each phase:

$$P = \sqrt{3}V_L I_L \cos\emptyset = \sqrt{3} \times 440 \times 11.54 \times 0.8 = 7035.7\text{ W} = 7.03\text{ kW Ans.}$$

That is the total power of three phase circuit. There are total three phases in the circuit. So power of each phase is

$$\text{POWER OF EACH PHASE} = \frac{P}{3} = \frac{7035.7}{3} = 2345.2\text{ W} = 2.34\text{ kW } \textbf{Ans}.$$