UNIT 2

ANALYSIS OF SINGLE PHASE AC CIRCUIT

3.1 BASIC AC CIRCUIT

There are three basic AC circuit

3.1.1 **Purely resistive circuit:** This circuit only consist resistor as shown in fig. 3.1

The voltage applied to resistor is

$$v = V_m \sin \omega t \qquad ....(3.1)$$

The current in resistor is given by

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} \qquad ....(3.2)$$

This current is maximum when $\sin \omega t = 1$. The maximum current is

$$I_m = \frac{V_m}{R}$$

Substitute the value of I_m in eq. (3.2)

$$i = I_m \sin \omega t \qquad ....(3.3)$$

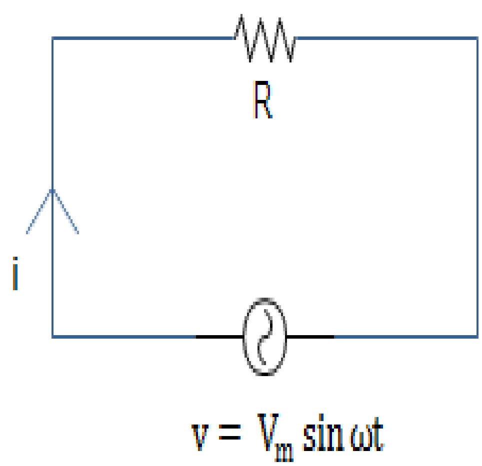


Fig. 3.1

By comparing the equation 3.1 and 3.3 of voltage and current we can say in purely resistive circuit voltage and current are in same phase.

Phase Angle: It is the angle between the supply voltage and supply current. It is denoted by $\emptyset$.

In purely resistive circuit phase angle $\emptyset = 0°$

The wave form diagram and phasor diagram for purely resistive circuit is shown in fig.3.2 (a) and fig. 3.2(b)

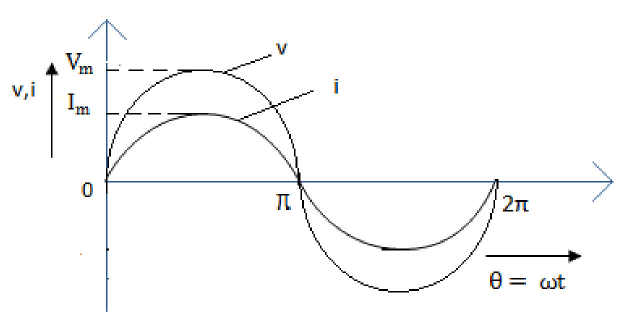


Fig. 3.2 (a)

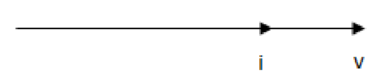


Fig. 3.2 (b)

Power in purely resistive circuit is given by equation

$$p = v \times i = V_m \sin \omega t \times I_m \sin \omega t = V_m I_m \sin^2 \omega t$$

The average value of power is

$$P_{av} = \frac{\int_0^{\pi} p\, d\theta}{\pi} = \frac{\int_0^{\pi} V_m I_m \sin^2 \omega t\, d\omega t}{\pi} = \frac{V_m I_m}{\pi} \int_0^{\pi} \left(\frac{1-\cos 2\omega t}{2}\right) d\omega t = \frac{V_m I_m}{2\pi}\left(\theta - \frac{\sin 2\omega t}{2}\right)_0^{\pi}$$

$$= \frac{V_m I_m}{2\pi}\left(\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right) = \frac{V_m I_m(\pi)}{2\pi} = \frac{V_m I_m}{2} = \frac{V_m I_m}{\sqrt{2}\sqrt{2}}$$

$$P_{av} = V_{rms} I_{rms} = VI = I^2 R = \frac{V^2}{R}$$

3.1.2 **Purely inductive circuit:** This circuit only consist inductor as shown in fig. 3.3

The voltage applied to inductor is

$$v = V_m \sin \omega t \qquad ....(3.4)$$

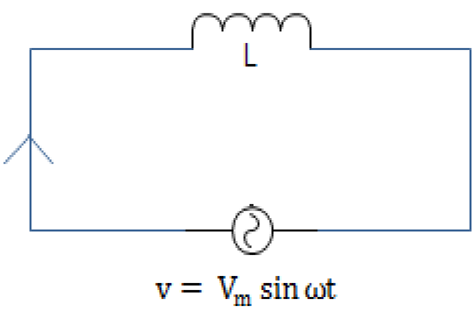


Fig. 3.3

The current in inductor is given by

$$i = \frac{1}{L}\int v\, dt = \frac{1}{L}\int V_m \sin \omega t\ dt = \frac{-V_m \cos \omega t}{\omega L}$$

$$= \frac{-V_m \sin(90° - \omega t)}{\omega L}$$

$$= \frac{V_m \sin(\omega t - 90°)}{\omega L}$$

Where $\omega L = 2\pi f L = X_L$ is called inductive reactance. It is the opposition offered by the inductor to the flow of current. The unit of the X_L is ohm (Ω). So

$$i = \frac{V_m \sin(\omega t - 90°)}{X_L} \qquad ....(3.5)$$

This current is maximum when $\sin(\omega t - 90) = 1$. The maximum current is

$$I_m = \frac{V_m}{X_L}$$

Substitute the value of I_m in eq. (3.5),we get

$$i = I_m \sin(\omega t - 90°) \qquad ....(3.6)$$

By comparing the equation 3.4 and 3.6 of voltage and current we can say in purely inductive circuit current is lagging from voltage by an angle 90°(π/2).

Phase angle for purely inductive circuit

$$\emptyset = 90°(\text{lagging})$$

The wave form diagram and phasor diagram for purely inductive circuit is shown in fig. 3.4 (a) and fig. 3.4 (b) respectively.

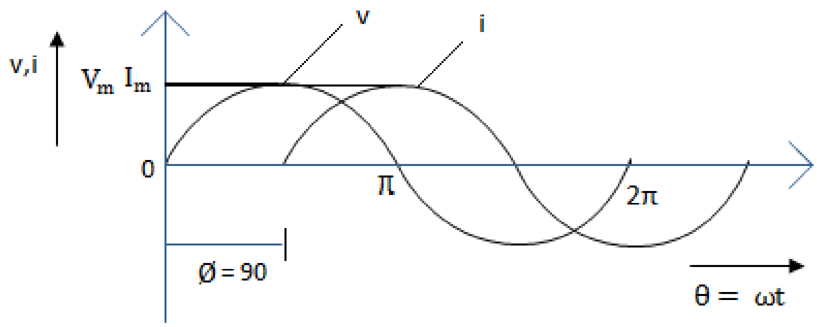


Fig. 3.4 (a)

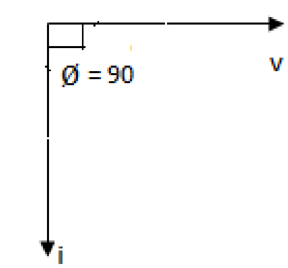


Fig. 3.4 (b)

Power in purely resistive circuit is given by equation

$$p = v \times i = V_m \sin \omega t \times I_m \sin(\omega t - 90) = -V_m I_m \sin \omega t \cos \omega t = \frac{-V_m I_m \sin 2\omega t}{2}$$

The average value of power is

$$P_{av} = \frac{\int_0^{\pi} p\, d\theta}{\pi} = \frac{\int_0^{\pi} \left(\frac{-V_m I_m \sin 2\omega t}{2}\right) d\omega t}{\pi} = \frac{V_m I_m}{2\pi}\left(\frac{\cos 2\omega t}{2}\right)_0^{\pi} = \frac{V_m I_m}{2\pi}\left(\frac{\cos 2\pi}{2} - \frac{\cos 0}{2}\right)$$

$$= \frac{V_m I_m (1-1)}{4\pi} = 0$$

$$P_{av} = 0$$

3.1.3 **Purely capacitive circuit:** This circuit only consist capacitor as shown in fig. 3.5

The voltage applied to capacitor is

$$v = V_m \sin \omega t \qquad \text{....(3.7)}$$

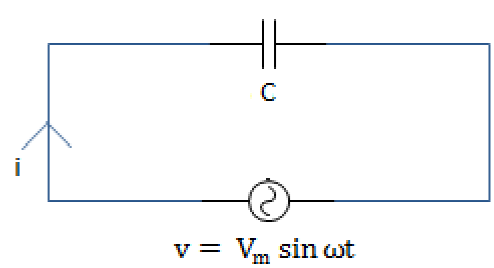


Fig. 3.5

The current in capacitor is given by

$$i = \frac{C\,dv}{dt} = \frac{C\,dV_m \sin \omega t}{dt} = \omega C V_m \cos \omega t$$

$$= \frac{V_m \cos \omega t}{(1/\omega C)}$$

$$= \frac{V_m \sin(90° + \omega t)}{(1/\omega C)}$$

$$= \frac{V_m \sin(\omega t + 90°)}{(1/\omega C)}$$

Where $\frac{1}{\omega C} = \frac{1}{2\pi f C} = X_C$ is called capacitive reactance. It is the opposition offered by the capacitor to the flow of current. The unit of the X_C is ohm (Ω).so

$$i = \frac{V_m \sin(\omega t + 90°)}{X_C} \quad \text{....(3.8)}$$

This current is maximum when $\sin(\omega t + 90°) = 1$. The maximum current is

$$I_m = \frac{V_m}{X_C}$$

Substitute the value of I_m in eq. (3.8),we get

$$i = I_m \sin(\omega t + 90°)$$

....(3.9)

By comparing the equation of voltage and current we can say in purely capacitive circuit current is leading from voltage by an angle 90°(π/2).

Phase angle for purely capacitive circuit $\emptyset = 90°$(leading)

The wave form diagram and phasor diagram for purely capacitive circuit is shown in fig. 1.6 (a) and (b)

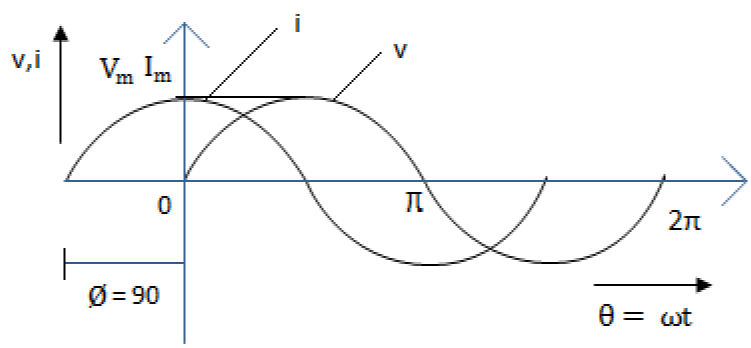


Fig. 3.6 (a)

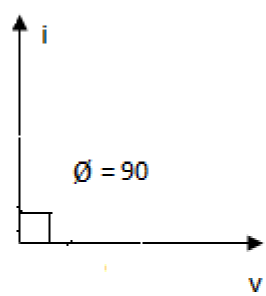


Fig. 3.6 (b)

Power in purely capacitive circuit is given by equation

$$p = v \times i = V_m \sin\omega t \times I_m \sin(\omega t + 90) = V_m I_m \sin\omega t \cos\omega t = \frac{V_m I_m \sin 2\omega t}{2}$$

The average value of power is

$$P_{av} = \frac{\int_0^{\pi} p \, d\theta}{\pi} = \frac{\int_0^{\pi} \left(\frac{V_m I_m \sin 2\omega t}{2}\right) d\omega t}{\pi} = \frac{V_m I_m}{2\pi} \left(\frac{-\cos 2\omega t}{2}\right)_0^{\pi}$$

$$= \frac{-V_m I_m}{2\pi}\left(\frac{\cos 2\pi}{2} - \frac{\cos 0}{2}\right) = \frac{-V_m I_m (1-1)}{2\pi} = 0$$

$$P_{av} = 0$$

3.2 SERIES AC CIRCUIT

In series AC circuit R, L or C are connected in series. Series AC circuit are of three types.

3.2.1 Series R-L circuit: The circuit diagram and phasor diagram of series R-L circuit is shown in fig. 3.7 (a) and fig. 3.7 (b) respectively.

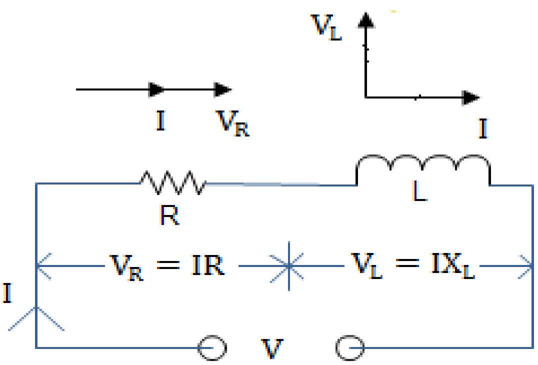


Fig. 3.7 (a)

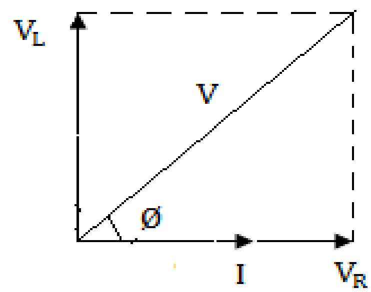


Fig. 3.7 (b)

Where

V = RMS value of supply voltage

I = RMS value of supply current

V_R = RMS value of voltage across resistance

V_L = RMS value of voltage across inductance L

From the phasor diagram shown in fig. 3.7 (b)

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + {X_L}^2}$$

Where $\sqrt{R^2 + {X_L}^2} = Z$ is called the impedance of the series R-L circuit. . It is the opposition offered by the series R-L circuit to the flow of current. The unit of the Z is ohm (Ω).

So the relation between supply voltage, supply current and impedance is

$$V = IZ$$

From the phasor diagram shown in fig. 3.7 (b)

$$\tan \emptyset = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$$

So the phase angle

$$\emptyset = \tan^{-1}\left(\frac{X_L}{R}\right)$$

From the phasor diagam shown in fig. 3.7 (b) we can say in series R-L circuit supply current is lagging from supply voltage by an angle $\emptyset$.

Phase angle for R-L circuit $0 < \emptyset < 90°$ (lagging)

3.2.2 Series R-C circuit: The circuit diagram and phasor diagram of series R-C circuit is shown in fig. 3.8 (a) and fig. 3.8 (b) respectively.

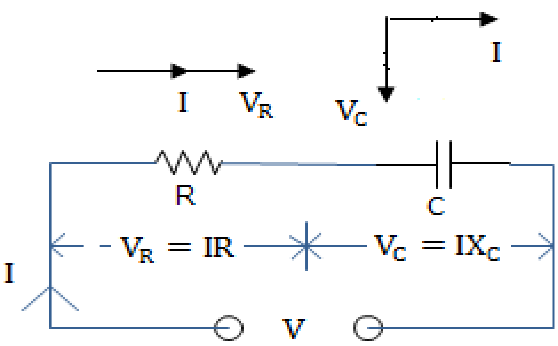


Fig. 3.8 (a)

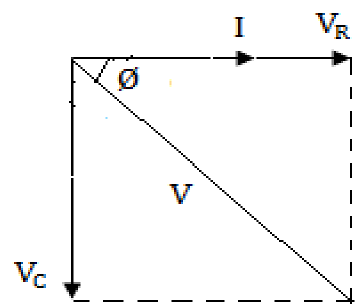


Fig 3.8 (b)

Where

V = RMS value of supply voltage

I = RMS value of supply current

V_R = RMS value of voltage across resistance

V_C = RMS value of voltage across capacitance C

From the phasor diagram shown in fig. 3.8 (b)

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2 + {X_C}^2}$$

Where $\sqrt{R^2 + {X_C}^2} = Z$ is called the impedance of the series R-C circuit. . It is the opposition offered by the series R-C circuit to the flow of current. The unit of the Z is ohm (Ω).

So the relation between supply voltage, supply current and impedance is

$$V = IZ$$

From the phasor diagram shown in fig. 3.8 (b)

$$\tan\emptyset = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R}$$

So the phase angle

$$\emptyset = \tan^{-1}\left(\frac{X_C}{R}\right)$$

From the phasor diagam shown in fig. 3.8 (b) we can say in series R-C circuit supply current is leading from supply voltage by an angle Ø.

Phase angle for R-L circuit $0 < \emptyset < 90°$ (leading)

3.2.3 Series R-L-C circuit: The circuit diagram and phasor diagram of series R-L-C circuit is shown in fig. 3.9 (a) and fig. 3.9 (b) respectively.

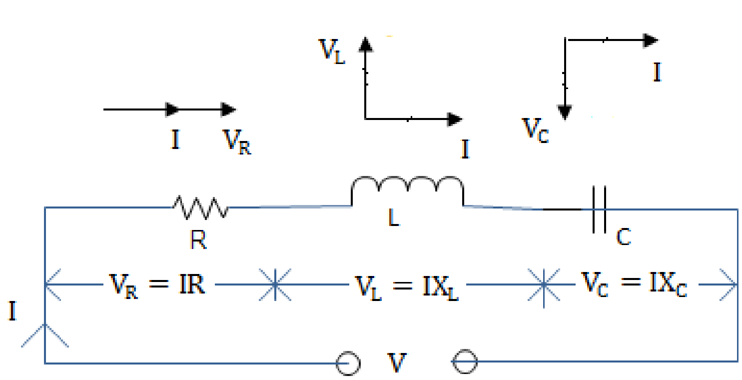


Fig. 3.9 (a)

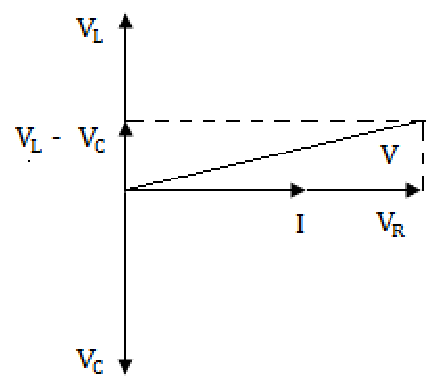


Fig. 3.9 (b)

Where

V = RMS value of supply voltage

I = RMS value of supply current

V_R = RMS value of voltage across resistance

V_L = RMS value of voltage across inductance

V_C = RMS value of voltage across capacitor C

From the phasor diagram shown in fig.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$$

Where $\sqrt{R^2 + (X_L - X_C)^2} = Z$ is called the impedance of the series R-L-C circuit. . It is the opposition offered by the series R-L-C circuit to the flow of current. The unit of the Z is ohm (Ω).

So the relation between supply voltage, supply current and impedance is

$$V = IZ$$

From the phasor diagram shown in fig. 3.9 (b)

$$\tan\emptyset = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$

So the phase angle

$$\emptyset = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

The phase angle for series R-L-C circuit is $0 < \emptyset < 90°$

If $X_L = X_C$ then R-L-C circuit will be resistive. Current is in same phase with voltage. $\emptyset = 0°$

If $X_L > X_C$ then R-L-C circuit will be inductive. Current is lagging from voltage. $\emptyset = \text{positive}$

If $X_L < X_C$ then R-L-C circuit will be capacitive. Current is leading from voltage. $\emptyset = \text{negative}$

3.3 POWER IN AC CIRCUIT

There are three type of power in AC circuit.

3.3.1 **Active power (Real power/True power/Total power):** It is the product of the RMS value of voltage, current and cosine of the phase angle between voltage and current (Ø). It is denoted by P and unit is Watt.

$$P = VI\cos\emptyset \text{ (Watt)}$$

3.3.2 **Reactive power:** It is the product of the RMS value of voltage, current and sine of the phase angle between voltage and current (Ø). It is denoted by Q and unit is Volt Ampere Reactive (VAR).

$$Q = VI\sin\emptyset \text{ (VAR)}$$

3.3.3 **Apparent power:** It is the product of the RMS value of voltage, current. It is denoted by S and unit is Volt Ampere (VA)

$$S = VI \text{ (VA)}$$

3.4 POWER FACTOR

Cosine of the angle between voltage and current is called the power factor.

$$\text{p.f.} = \cos\emptyset = \frac{R}{Z} = \frac{P}{S}$$

For pure resistive circuit phase angle $\emptyset = 0$, power factor $(\cos\emptyset) = 1$
For pure inductive circuit phase angle $\emptyset = 90(\text{lagging})$, power factor $(\cos\emptyset) = 0(\text{lagging})$
For pure capacitive circuit phase angle $\emptyset = 0(\text{leading})$, power factor $(\cos\emptyset) = 0(\text{leading})$
For series R-L circuit phase angle is $0 < \emptyset < 90, power\ factor\ is\ 0 < \emptyset < 1(\text{lagging})$
For series R-C circuit phase angle is $0 < \emptyset < 90$, power factor is $0 < \emptyset < 1(\text{leading})$

3.5 IMPEDANCE TRIANGLE AND POWER TRIANGLE

The triangle between R, X_L, X_C and Z is called the impedance triangle as shown in fig. 3.10 (a)
The triangle between P, Q and S is called the power triangle as shown in fig. 3.10 (b)

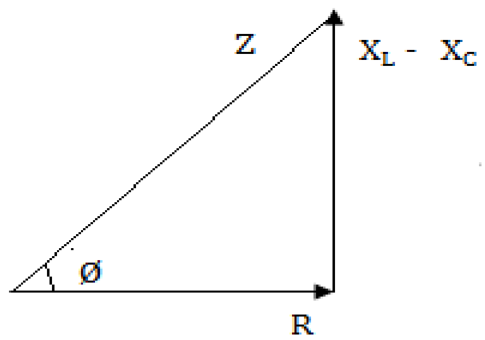


Fig 3.10 (a)

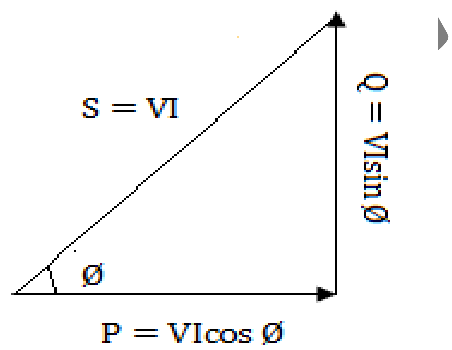


Fig. 3.10 (b)

3.6 FORMULA USE FOR NUMERICAL OF SINGLE PHASE AC CIRCUIT

All the calculation in the numerical of AC circuit is for RMS value.

$$\text{inductive reactance } X_L(\Omega) = 2\pi f L(\text{inductance})$$

$$\text{capacitive reactance } X_C(\Omega) = 1/2\pi f C(\text{capacitance})$$

For any AC circuit relation use between supply voltage, supply current and impedance is

$$V = IZ$$

Now the value of Z will depend on the type of the circuit

For purely R circuit $Z = R$

For purely L circuit $Z = X_L$

For purely C circuit $Z = X_L$

For series R-L circuit $Z = \sqrt{R^2 + {X_L}^2}$

For series R-C circuit $Z = \sqrt{R^2 + {X_C}^2}$

For series R-L-C circuit $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Active power (Real power/True power/Total power) $P = VI\cos\emptyset$ (Watt)

Reactive power $Q = VI\sin\emptyset$ (VAR)

Apparent power $S = VI$ (VA)

$$\text{Power Factor} = \cos\emptyset = \frac{R}{Z} = \frac{P}{S}$$

Example 3.1 The voltage and current through a circuit are $v = 50\sin(500t + 55)$ and $i = 5\sin(500t + 325)$.Find (1) Identify the circuit element (2) Find the value of circuit element (3) Power

Solution: From the equation of voltage and current we get the following information

$$V_m = 50\text{ V}, I_m = 5\text{ Amp.}, \omega = 500$$

By drawing the phasor diagram shown in fig. 3.11, We can say $\emptyset = 90$, So

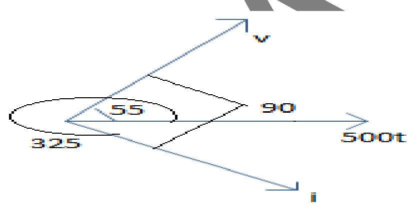


Fig. 3.11

(1) The circuit is purely inductive. **Ans.**
(2) The relation between V, I and Z is $V = IZ$

$$V = \frac{V_m}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.35\text{ V and } I = \frac{I_m}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.53\text{ A}$$

$$Z = \frac{V}{I} = \frac{35.35}{3.53} = 10\ \Omega$$

In purely inductive circuit $Z = X_L = \omega L = 10$

$$L = \frac{10}{\omega} = \frac{10}{500} = 0.02\text{ H Ans.}$$

(3) Power $P = VI\cos\emptyset = = 0\text{ W}$ **Ans.**

Example 3.2 The voltage and current through a circuit are $v = 100\sin(400t)$ and $i = 10\sin(400t - 30)$.Find (1) Identify the circuit element (2) Impedance (3) Active Power (4) Reactive Power (5) Apparent Power (6) Power factor

Solution: From the equation of voltage and current we get the following information

$$V_m = 100\text{ V}, I_m = 10\text{ Amp.}, \omega = 400$$

By drawing the phasor diagram shown in fig. 3.12

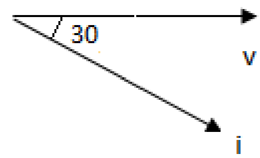


Fig. 3.12

From the phasor diagram we can say the current is lagging from voltage by an angle $\emptyset = 30°$ so

(1) The circuit is series R-L circuit. **Ans.**

(2) The relation between V, I and Z is $V = IZ$

$$V = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71\ \text{V and}\ I = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07\ \text{A}$$

$$Z = \frac{V}{I} = \frac{70.71}{7.07} = 10\ \Omega\ \textbf{Ans}.$$

(3) Active Power $P = VI\cos\emptyset = 70.71 \times 7.07 \times \cos 30° = 432.94\ \text{W}$ **Ans**.

(4) Reactive Power $Q = VI\sin\emptyset = 70.71 \times 7.07 \times \sin 30° = 250\ \text{VAR}$ **Ans**.

(5) Apparent Power $S = VI = 70.71 \times 7.07 = 500\ \text{VA}$ **Ans**.

(6) Power factor = $\cos\emptyset = \cos 30 = 0.866$ (lagging) **Ans**.

Example 3.3 The voltage and current through a circuit are $v = 10\sqrt{2}\sin(314t)$ and $i = 5\sqrt{2}\sin(314t + 60)$.Find (1) Identify the circuit element (2) Impedance (3) Active Power (4) Reactive Power (5) Apparent Power (6) Power factor

Solution: From the equation of voltage and current we get the following information

$$V_m = 10\sqrt{2}\ \text{V}, I_m = 5\sqrt{2}\ \text{Amp.}, \omega = 400$$

By drawing the phasor diagram shown in fig.

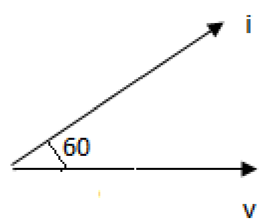


Fig. 3.13

From the phasor diagram we can say the current is leading from voltage by an angle $\emptyset = 60$ so

(1) The circuit is series R-C circuit. **Ans.**

(2) The relation between V, I and Z is $V = IZ$

$$V = \frac{V_m}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = 10\ \text{V and}\ I = \frac{I_m}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}} = 5\ \text{Amp.}$$

$$Z = \frac{V}{I} = \frac{10}{5} = 2\,\Omega\ \textbf{Ans.}$$

(3) Active Power $P = VI\cos\emptyset = 10 \times 5 \times \cos 60 = 25\ W$ **Ans.**

(4) Reactive Power $P = VI\sin\emptyset = 10 \times 5 \times \sin 60 = 43.3\ VAR$ **Ans.**

(5) Apparent Power $P = VI = 10 \times 5 = 50\ VA$ **Ans.**

(6) Power factor = $\cos\emptyset = \cos 60 = 0.5\ (\text{leading})$ **Ans.**

Example 3.4 A non inductive resistance of 10 Ω is connected in series with an inductive coil across 200 V, 50 Hz supply. The current drawn by the series combination is 10 Amp. The resistance of the coil is 2 Ω. Determine (1) Inductance of the coil (2) Power factor (3) Voltage across the non inductive resistance (4) Voltage across the coil

Solution: The non inductive resistance is pure resistance and inductive coil is the series R-L circuit. The complete circuit is shown in fig. 3.14

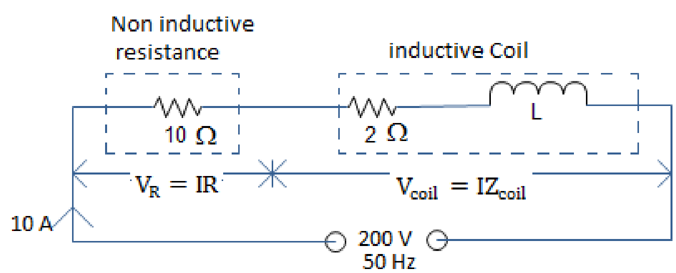


Fig. 3.14

(1) The relation between V, I and Z is $V = IZ$

$$Z = \frac{V}{I} = \frac{200}{10} = 20\,\Omega$$

The impedance of the R-L circuit is

$$Z = \sqrt{R^2 + {X_L}^2} \Rightarrow 20 = \sqrt{12^2 + {X_L}^2} \Rightarrow X_L = 16\,\Omega$$

We know

$$X_L = 2\pi f L \Rightarrow L = \frac{X_L}{2\pi f} = \frac{16}{2 \times \pi \times 50} = 0.051 = 51\ mH\ \textbf{Ans.}$$

(2) Power Factor

$$\cos\emptyset = \frac{R}{Z} = \frac{12}{20} = 0.6\ (\text{lagging})\textbf{Ans.}$$

(3) Voltage across the non inductive resistance(Pure resistance)

$$V_R = IR = 10 \times 10 = 100\ V\ \textbf{Ans.}$$

(4) Voltage across the coil

$$V_{coil} = IZ_{coil} = I \times \sqrt{R_{coil}^2 + X_{Lcoil}^2} = 10 \times \sqrt{2^2 + 16^2} = 161.24\ V\ \textbf{Ans.}$$

Example 3.5 A 100 V, 200 W lamp is to be operated on 220 V, 50 Hz supply. In order that lamp should operate on correct voltage. Calculate the value of (1) Non inductive resistance (2) Pure inductance (3) Pure capacitance

Solution: The Lamp, bulb and press this type of apparatus is considered purely resistive circuit as shown in fig. 3.15

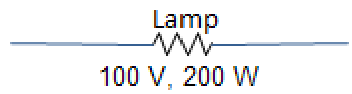


Fig. 3.15

The current in the lamp

$$P = VI \Rightarrow I = \frac{P}{V} = \frac{200}{100} = 2 \text{ Amp.}$$

The resistance of the lamp

$$V = IR_{lamp} = R_{lamp} = \frac{V}{I} = \frac{100}{2} = 50\ \Omega$$

The supply voltage is 220 V but the voltage rating of the lamp is 100 V. To operate the lamp on correct voltage we have to connect an element(R, L, C) in series with the lamp.

(1) The non inductive resistance (pure resistance): In this case the circuit is purely resistive as shown in fig. 3.16

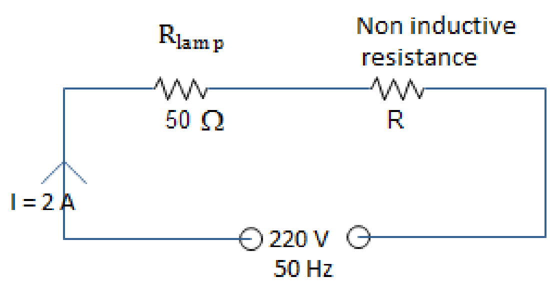


Fig. 3.16

The relation between V , I and Z is

$$V = IZ \Rightarrow Z = \frac{V}{I} = \frac{220}{2} = 110\ \Omega$$

The circuit is purely resistive so $Z = R_{lamp} + R \Rightarrow 110 = 50 + R \Rightarrow R = 60\ \Omega$ **Ans.**

(2) Pure inductance: In this case the circuit is series R-L as shown in fig. 3.17

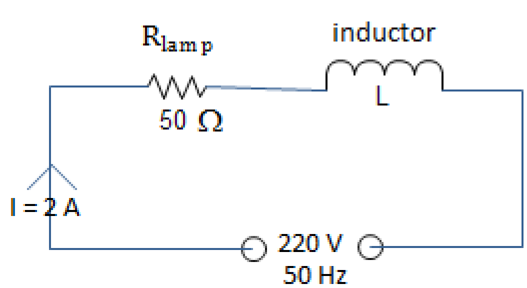


Fig. 3.17

The relation between V, I and Z is

$$V = IZ \Rightarrow Z = \frac{V}{I} = \frac{220}{2} = 110\,\Omega$$

Now the circuit is series R-L so

$$Z = \sqrt{R_{lamp}^2 + X_L^2} \Rightarrow 110 = \sqrt{50^2 + X_L^2} \Rightarrow X_L = 100$$

We know

$$X_L = 2\pi f L \Rightarrow L = \frac{X_L}{2\pi f} = \frac{100}{2 \times \pi \times 50} = 0.318 = 318\,\text{mH}\ \textbf{Ans.}$$

(3) Pure capacitance: In this case the circuit is series R-C as shown in fig. 3.18

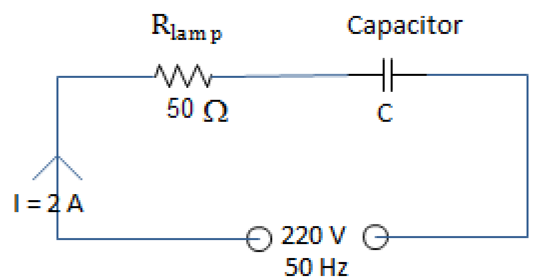


Fig. 3.18

The relation between V, I and Z is

$$V = IZ \Rightarrow Z = \frac{V}{I} = \frac{220}{2} = 110\,\Omega$$

Now the circuit is series R-L so

$$Z = \sqrt{R_{lamp}^2 + X_C^2} \Rightarrow 110 = \sqrt{50^2 + X_C^2} \Rightarrow X_C = 100$$

We know

$$X_c = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi f X_c} = \frac{1}{2 \times \pi \times 50 \times 100} = 31.8 \times 10^{-6} = 31.8\,\mu\text{F}\ \textbf{Ans.}$$

Example 3.6 An iron coil takes 3 A when connected to a 15 V DC supply and takes 5 A from 65 V, 50 Hz AC supply, determine (1) Resistance and inductance of the coil (2) Power factor (3) Power drawn by the coil

Solution: Two supplies are given to a coil DC and AC as shown in fig. 3.19 (a) and 3.19 (b) respectively.

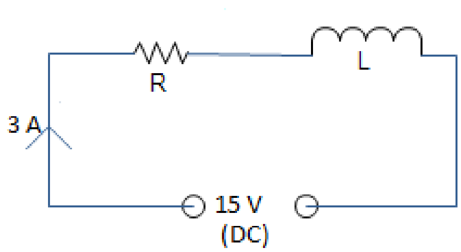


Fig. 3.19 (a)

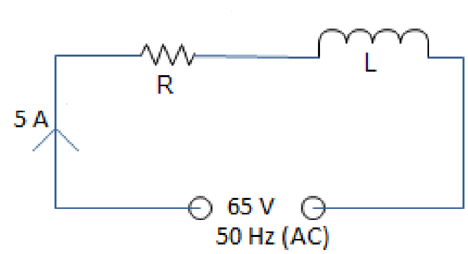


Fig 3.19 (b)

In the case of DC the circuit is behave like purely resistive circuit, because for DC frequency f =0 so

$$X_L = 2\pi f L = 2 \times \pi \times 0 \times L = 0$$

So inductor will behave like short circuit.

(1) Resistance and inductance of the coil:

From DC circuit

$$R = \frac{V}{I} = \frac{15}{3} = 5\ \Omega\ \textbf{Ans.}$$

From AC circuit

$$Z = \frac{V}{I} = \frac{65}{5} = 13\ \Omega$$

For series R-L circuit

$$Z = \sqrt{R^2 + X_L^2} \Rightarrow 13 = \sqrt{5^2 + X_L^2} \Rightarrow X_L = 12\ \Omega$$

We know

$$X_L = 2\pi f L \Rightarrow L = \frac{X_L}{2\pi f} = \frac{12}{2 \times \pi \times 50} = 0.0382 = 38.2\ \text{mH}\ \textbf{Ans.}$$

(2) Power factor

$$\cos\emptyset = \frac{R}{Z} = \frac{5}{13} = 0.384(\text{lagging})\ \textbf{Ans.}$$

(3) Power drawn by the coil

$$P = VI\cos\emptyset = 65 \times 5 \times 0.384 = 124.8\ \text{W}\ \textbf{Ans.}$$

Example 3.7 A coil of resistance 10 Ω and inductance 0.15 H are connected in series with a condenser of capacitance 120 μF across 230 V, 50 Hz supply. Determine (1) Impedance (2) Current (3) Power factor (4) Voltage across coil (5) Voltage across condenser

Solution: The complete circuit is shown in fig. 3.20

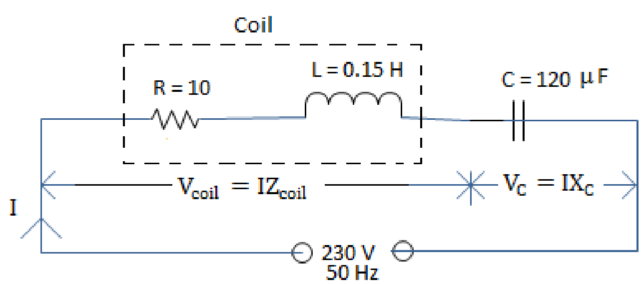


Fig. 3.20

$$X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.15 = 47.12\ \Omega$$

$$X_c = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 50 \times 120 \times 10^{-6}} = 26.53\ \Omega$$

(1) Impedance: For series R-L-C circuit the impedance is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{10^2 + (47.12 - 26.53)^2} = 22.9\ \Omega\ \textbf{Ans.}$$

(2) Current: We know

$$I = \frac{V}{Z} = \frac{230}{22.9} = 10.04 \text{ Amp. } \textbf{Ans.}$$

(3) Power Factor: We know

$$\cos\emptyset = \frac{R}{Z} = \frac{10}{22.9} = 0.4367 \textbf{ Ans.}$$

(4) Voltage across coil: We know

$$V_{coil} = IZ_{coil} = I \times \sqrt{R_{coil}^2 + X_{Lcoil}^2} = 10.04 \times \sqrt{10^2 + 47.12^2} = 483.62 \text{ V } \textbf{Ans.}$$

(5) Voltage across condenser: We know

$$V_c = IX_c = 10.04 \times 26.53 = 266.36 \text{ V } \textbf{Ans.}$$

Example 3.8 A resistance (R) and inductance L = 0.05 H and a capacitance C are connected in series. When a voltage $v = 40\sin(5000t - 15)$ is applies to series combination the current flowing is and $i = 5\sin(5000t - 70)$.Find (1) Resistance (2) Capacitance

Solution: The circuit is shown in fig. 3.21

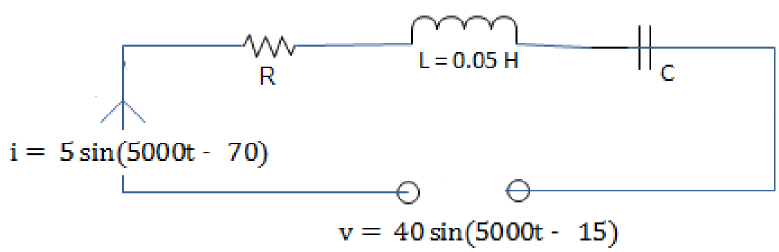


Fig. 3.21

Form the eq. of voltage and current we get $V_m = 40$ V, $I_m = 5$ A, $\omega = 5000$, $\emptyset = 55°$(lag)

So

$$V = \frac{V_m}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 28.28 \text{ V} \qquad I = \frac{I_m}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.54 \text{ A}$$

$$X_L = \omega L = 5000 \times 0.05 = 250\ \Omega$$

$$Z = \frac{V}{I} = \frac{28.28}{3.54} = 8\ \Omega$$

(1) Resistance (R): We know

$$\cos\emptyset = \frac{R}{Z} \Rightarrow \cos 55° = \frac{R}{8} \Rightarrow R = 4.59\ \Omega \textbf{ Ans.}$$

(2) Capacitance (C): We know for series R-L-C circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \Rightarrow 8 = \sqrt{4.59^2 + (250 - X_C)^2} \Rightarrow X_C = 243.45\ \Omega$$

$$X_c = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_c} = \frac{1}{5000 \times 243.45} = 8.2 \times 10^{-7} = 0.82\ \mu\text{F } \textbf{Ans.}$$

3.7 PARALLEL AC CIRCUIT

In parallel AC circuit R, L or C are connected in parallel. For solving parallel RLC circuit we will use the complex form of the impedance.

If R-L-C is connected in series then complex form of the impedance is

$$Z = R + j(X_L - X_C)$$

For purely R circuit $Z = R$

For purely L circuit $Z = jX_L$

For purely C circuit $Z = -jX_C$

For series R-L circuit $Z = R + jX_L$

For series R-C circuit $Z = R - jX_C$

For series R-L-C circuit $Z = R + j(X_L - X_C)$

3.8 COMPLEX NUMEBER

For solving parallel RLC circuit we required knowledge of solving complex number. There are two form of complex number

3.8.1 **Rectangular form:** It is represented by

$$a + jb$$

Where a = length of the real axis of the complex plane
b = length of the imaginary axis of the complex plane

3.8.2 **Polar form:** It is represented by

$$r\angle\theta$$

Where r = length of the complex number from origin
θ = Angle of complex number of from the x axis

3.9 CONVERSION OF FORM

Conversion from rectangular form to polar form $(\mathbf{a + jb \rightarrow r\angle\theta})$**:**

$$r = \sqrt{a^2 + b^2} \qquad \theta = \tan^{-1}\frac{b}{a}$$

Conversion from rectangular form to polar form $(\mathbf{r\angle\theta \rightarrow a + jb})$**:**

$$a = r\cos\theta \qquad b = r\sin\theta$$

Example 3.9 Convert 4+j3 in polar form

Solution: $4 + j3 \rightarrow r\angle\theta$

$$r = \sqrt{4^2 + 3^2} = 5 \qquad \theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{3}{4} =$$
$$4 + j3 \rightarrow 5\angle\theta \ \textbf{Ans}.$$

Example 3.10 Convert $10\angle 60$ in rectangular form

Solution: $10\angle 60 \rightarrow a + jb$

$$a = 10\cos 60 = 5 \qquad b = 10\sin 60 = 8.66$$
$$10\angle 60 \rightarrow 5 + j8.66$$

Example 3.11 Perform the addition, subtraction, multiplication and division of 5+j4 and 3+j8

Solution: Addition $(5 + j4) + (3 + j8) = 8 + j12$

Subtraction $(5 + j4) - (3 + j8) = 2 - j4$

Multiplication $(5 + j4) \times (3 + j8) = 15 + j40 + j12 - 32 = -17 + j52$

Division

$$\frac{(5 + j4)}{(3 + j8)} = \frac{(5 + j4)}{(3 + j8)} \times \frac{(3 - j8)}{(3 - j8)} = \frac{15 - j40 + j12 + 32}{9 - j24 + j24 + 64} = \frac{47 - j28}{73} = 0.64 - j0.383$$

Example 3.12 Perform the addition, subtraction, multiplication and division of $10\angle 30$ and $5\angle 60$

Solution: Addition and Subtraction is not possible with polar form. For that first you have to convert the polar form into rectangular form.

Multiplication $10\angle 30 \times 5\angle 60 = 50\angle 90$

Division $\frac{10\angle 30}{5\angle 60} = 2\angle -30$

<u>**Note:**</u> Addition and Subtraction are easy with rectangular form and Multiplication and division are easy with polar form

3.10 ADMITTANCE

Admittance is the reciprocal of the impedance. It is denoted by Y ant unit is mho ℧.

$$Y = \frac{1}{Z}$$

Example 3.13 For the circuit shown in fig. 3.22 Find (1) Total Impedance (2) Total Current (3) Current in each branch (4) Total Admittance (5) Admittance of each branch (6) Power factor (7) Active power (8) Reactive power (9) Apparent power (10) Draw the phasor diagram

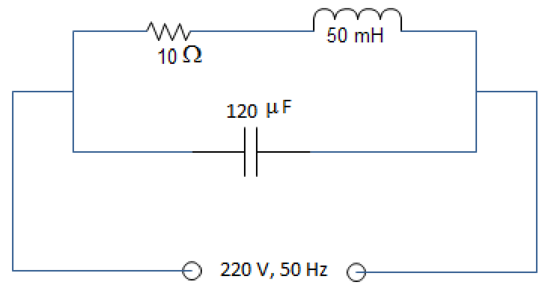


Fig. 3.22

Solution:

$$X_L = 2\pi fL = 2 \times \pi \times 50 \times 50 \times 10^{-3} = 15.7\ \Omega$$

$$X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 50 \times 120 \times 10^{-6}} = 26.53\ \Omega$$

The first branch is the series R-L circuit so the impedance of the first branch is

$$Z_1 = R + jX_L = 10 + j15.7 = 18.61\angle 57.5°$$

The second branch is the purely capacitive circuit so the impedance of second branch

$$Z_2 = -jX_C = -j26.53 = 26.53\angle -90°$$

(1) Total Impedance(Z): The first branch and second branch are connected in parallel so

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(18.61\angle 57.5°)(26.53\angle -90°)}{10 + j15.7 - j26.53} = \frac{493.72\angle -32.5°}{10 - j10.83}$$

$$= \frac{493.72\angle -32.5°}{14.74\angle -47.28°} = 33.5\angle 14.78°\ \Omega\ \textbf{Ans.}$$

(2) Total current (I): We know

$$I = \frac{V}{Z} = \frac{220}{33.5\angle 14.78°} = 6.57\angle -14.78°\ \mathrm{A}\ \textbf{Ans.}$$

(3) Current in each branch:

Current in first branch (I_1)

$$I_1 = \frac{V}{Z_1} = \frac{220}{18.61\angle 57.5°} = 11.82\angle -57.5°\ \mathrm{A}\ \textbf{Ans.}$$

Current in second branch (I_2)

$$I_2 = \frac{V}{Z_2} = \frac{220}{26.53\angle -90°} = 8.29\angle 90°\ \mathrm{A}\ \ \textbf{Ans.}$$

(4) Total Admittance(Y):

$$Y = \frac{1}{Z} = \frac{1}{33.5\angle 14.78°} = 0.03\angle -14.78°\ \mho\ \textbf{Ans.}$$

(5) Admittance of each branch:

Admittance of first branch (Y_1)

$$Y_1 = \frac{1}{Z_1} = \frac{1}{18.61\angle 57.5°} = 0.054\angle -57.5° \, \mho \; \textbf{Ans.}$$

Admittance of second branch (Y_2)

$$Y_2 = \frac{1}{Z_2} = \frac{1}{26.53\angle -90°} = 0.0377\angle 90° \, \mho \; \textbf{Ans.}$$

(6) Power factor $(\cos \emptyset)$:

$$\cos \emptyset = \cos 14.78° = 0.967 \; \textbf{Ans.}$$

(7) Active Power (P):

$$P = VI \cos \emptyset = 220 \times 6.57 \times 0.967 = 1397.7 \text{ W} = 1.39 \text{ kW} \; \textbf{Ans.}$$

(8) Reactive Power (Q):

$$Q = VI \sin \emptyset = 220 \times 6.57 \times 0.255 = 368.57 \text{ VAR} = 0.368 \text{ kVAR} \; \textbf{Ans.}$$

(9) Apparent Power (S):

$$S = VI = 220 \times 6.57 = 1445.4 \text{ VA} = 1.445 \text{ kVA} \; \textbf{Ans.}$$

(10)Phasor diagram: The phasor diagram is shown in fig. 3.23

Example 3.15 For the circuit shown in fig. 3.24 Find (1) Total Impedance (2) Total Current (3) Current in each branch (4) Total Admittance (5) Admittance of each branch (6) Power factor (7) Active power (8) Reactive power (9) Apparent power (10) Draw the phasor diagram

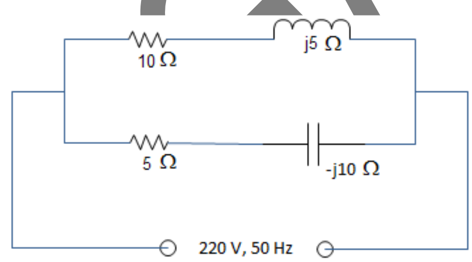


Fig. 3.24

Solution: The first branch is the series R-L circuit so the impedance of the first branch is

$$Z_1 = R + jX_L = 10 + j5 = 11.18\angle 26.56°$$

The second branch is the series R-C circuit so the impedance of second branch

$$Z_2 = R - jX_C = 5 - j10 = 11.18\angle -63.43°$$

(1) Total Impedance(Z): The first branch and second branch are connected in parallel so

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(11.18\angle 26.56°)(11.18\angle -63.43°)}{10 + j5 + 5 - j10} = \frac{125\angle -36.87°}{15 - j5}$$

$$= \frac{125\angle -36.87°}{15.81\angle -18.43°} = 7.9\angle -18.44° \, \Omega \; \textbf{Ans.}$$

(2) Total current (I): We know

$$I = \frac{V}{Z} = \frac{220}{7.9\angle -18.44°} = 27.85\angle 18.44° \text{ A } \textbf{Ans.}$$

(3) Current in each branch:

Current in first branch (I_1)

$$I_1 = \frac{V}{Z_1} = \frac{220}{11.18\angle 26.56°} = 19.68\angle -26.56° \text{ A } \textbf{Ans.}$$

Current in second branch (I_2)

$$I_2 = \frac{V}{Z_2} = \frac{220}{11.18\angle -63.43°} = 19.68\angle 63.43° \text{ A } \textbf{Ans.}$$

(4) Total Admittance(Y):

$$Y = \frac{1}{Z} = \frac{1}{7.9\angle -18.44°} = 0.126\angle 18.44° \text{ ℧ } \textbf{Ans.}$$

(5) Admittance of each branch:

Admittance of first branch (Y_1)

$$Y_1 = \frac{1}{Z_1} = \frac{1}{11.18\angle 26.56°} = 0.0894\angle -26.56° \text{ ℧ } \textbf{Ans.}$$

Admittance of second branch (Y_2)

$$Y_2 = \frac{1}{Z_2} = \frac{1}{11.18\angle -63.43°} = 0.0894\angle -63.43° \text{ ℧ } \textbf{Ans.}$$

(6) Power factor $(\cos \emptyset)$:

$$\cos \emptyset = \cos 18.44° = 0.9486 \textbf{ Ans.}$$

(7) Active Power (P):

$$P = VI\cos\emptyset = 220 \times 27.85 \times 0.9486 = 5812.07 \text{ W} = 5.81 \text{ kW } \textbf{Ans.}$$

(8) Reactive Power (Q):

$$Q = VI\sin\emptyset = 220 \times 27.85 \times 0.316 = 1936.13 \text{ VAR} = 1.93 \text{ kVAR } \textbf{Ans.}$$

(9) Apparent Power (S):

$$S = VI = 220 \times 27.85 = 6127 \text{ VA} = 6.12 \text{ kVA } \textbf{Ans.}$$

(10)Phasor diagram: The phasor diagram is shown is fig. 3.25

3.11 RESONANCE

Resonance is defined as a condition of the circuit containing at least one inductor and one capacitor then also the supply voltage and supply current are in same phase.

At the time of resonance power factor of the circuit is unity.$(\cos \emptyset = 1)$

Resonance is of two types

(1) Series Resonance (2) Parallel Resonance

3.12 SERIES RESONANCE

When resonance is create in series RLC circuit then it is called series resonance. A series RLC circuit and it phasor diagram under resonance condition is shown in fig. 3.26 (a) and 3.26 (b) respectively.

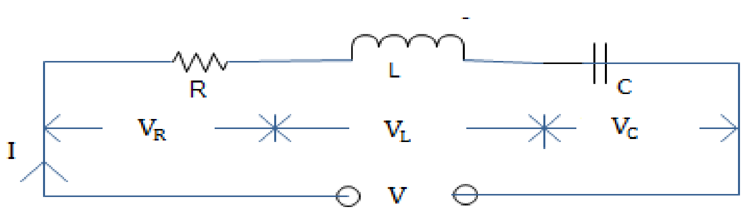


Fig. 3.26 (a)

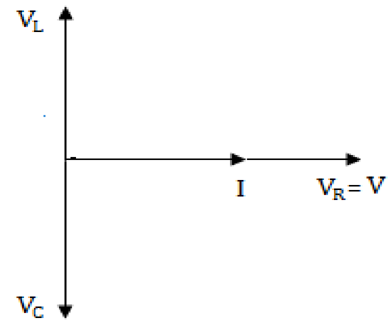


Fig. 3.26 (b)

Conditions of series resonance are

(1) $X_L = X_C$
(2) $V_L = V_C$
(3) Impedance Z = R (Minimum)
(4) Phase Angle $\emptyset = 0$
(5) Power Factor $\cos\emptyset = 1$
(6) Current I=(V/R) (Maximum)

Resonant Frequency: Frequency which creates resonance in the circuit is called resonant frequency. It is denoted by f_r. We have

$$X_L = X_C$$

$$\Rightarrow 2\pi f L = \frac{1}{2\pi f C} \Rightarrow f^2 = \frac{1}{(2\pi)^2 LC} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$$

Resonant frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Graphical representation of series resonance: Here we plot the graph between R, X_L, X_C, Z and I by varying the frequency.

(1) **Graph between R and f:** Resistance will not depend on the frequency, so the graph between R and f is shown in fig. 3.27 (a).
(2) **Graph between X_L and f:** We know $X_L = 2\pi f L \Rightarrow X_L \propto f$, So X_L is linearly increasing with the frequency. The graph between X_L *and* f is shown in fig. 3.27 (b).
(3) **Graph between X_C and f:** We know $X_C = 1/2\pi f C \Rightarrow X_C \propto 1/f$, so X_C is inversely proportional to the frequency. The graph between X_C and f is shown in fig. 3.27 (c).

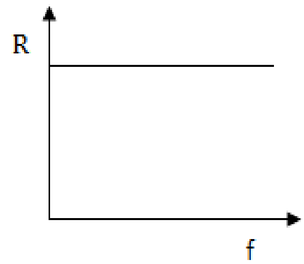


Fig. 3.27 (a)

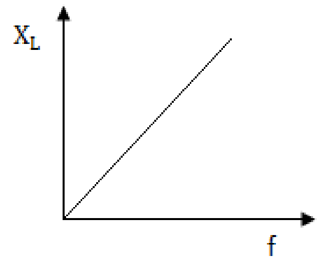


Fig. 3.27 (b)

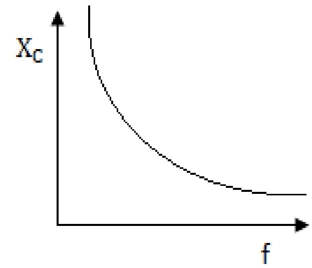


Fig. 3.27 (c)

(4) **Graph between Z and f:** We know $Z = \sqrt{R^2 + (X_L - X_C)^2}$, When $X_L = X_C$, then $Z = R$. If $X_L \neq X_C$,then $Z > R$. The graph between Z and f is shown in fig. 3.28 (a)

(5) **Graph between I and f:** We know $I = V/Z$, so the current is inversely proportional to the impedance. The graph between I and f is shown in fig.fig. 3.28 (b)

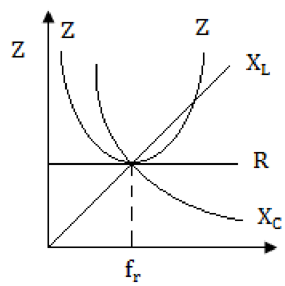


Fig. 3.28 (a)

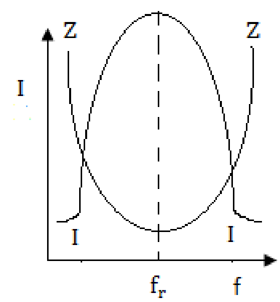


Fig. 3.28 (b)

Bandwidth: Bandwidth is defined as a range of frequency over which current is equal to or greater than the 70.7% of maximum current. From the graph shown in fig. 3.29.

Band width = $f_2 - f_1$

Where

f_1 is called the lower cut off frequency
f_2 is called the upper cut off frequency
f_1,f_2 is also called half power frequency

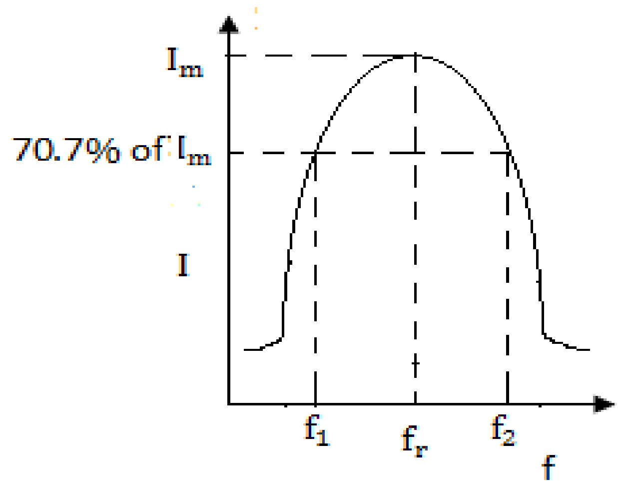


Fig. 3.29

Expression of Bandwidth: For the series RLC circuit the relation between supply voltage, supply current and Impedance is

$$V = IZ$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{R\sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}} = \frac{I_m}{\sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}}$$

At f_1 or f_2 the current is $\frac{I_m}{\sqrt{2}}$, so

$$\frac{I_m}{\sqrt{2}} = \frac{I_m}{\sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}} \Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}} \Rightarrow \left(\frac{X_L - X_C}{R}\right) = \pm 1$$

$$\Rightarrow X_L - X_C = \pm R$$

At f_1

$$X_L - X_C = -R$$

$$2\pi f_1 L - \frac{1}{2\pi f_1 C} = -R$$

....(3.10)

At f_2

$$X_L - X_C = R$$

$$2\pi f_2 L - \frac{1}{2\pi f_2 C} = R$$

....(3.11)

Add equation (3.10) with (3.11)

$$2\pi L(f_1 + f_2) - \frac{1}{2\pi C}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) = 0$$

$$\Rightarrow 2\pi L(f_1 + f_2) = \frac{1}{2\pi C}\left(\frac{f_1 + f_2}{f_1 f_2}\right)$$

$$\Rightarrow f_1 f_2 = \frac{1}{(2\pi)^2 LC}$$

We know

$$f_r = \frac{1}{2\pi\sqrt{LC}} \Rightarrow f_r^2 = \frac{1}{(2\pi)^2 LC}$$

So

$$f_1 f_2 = f_r^2$$

Subtract eq. (3.10) from (3.11)

$$2\pi L(f_2 - f_1) + \frac{1}{2\pi C}\left(\frac{1}{f_1} - \frac{1}{f_2}\right) = 2R$$

$$\Rightarrow 2\pi L(f_2 - f_1) + \frac{1}{2\pi C}\left(\frac{f_2 - f_1}{f_1 f_2}\right) = 2R$$

Substitute the value of $f_1 f_2$

$$(f_2 - f_1)\left[2\pi L + \frac{1}{2\pi C}\left(\frac{1}{\frac{1}{(2\pi)^2 LC}}\right)\right] = 2R$$

$$\Rightarrow (f_2 - f_1)[2\pi L + 2\pi L] = 2R$$

$$\Rightarrow (f_2 - f_1)(4\pi L) = 2R$$

$$\Rightarrow B.W. = (f_2 - f_1) = \frac{R}{2\pi L}$$

The lower cut off frequency

$$f_1 = f_r - \frac{R}{4\pi L}$$

The upper cut off frequency

$$f_2 = f_r + \frac{R}{4\pi L}$$

Quality Factor (Q.F.): It is defined as the ratio of voltage across the inductor to the supply voltage.

$$Q.F. = \frac{\text{Voltage across the inductor}}{\text{Supply voltage}} = \frac{V_L}{V} = \frac{IX_L}{IZ} = \frac{X_L}{Z} = \frac{2\pi fL}{Z}$$

At resonance $f = f_r$ and $Z = R$, so

$$Q.F. = \frac{2\pi f_r L}{R} = \frac{2\pi L}{R} \times \frac{1}{2\pi\sqrt{LC}} = \frac{L}{R\sqrt{LC}}$$

$$Q.F. = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Relation between resonance frequency (f_r), Bandwidth (B.W.) and Quality factor (Q.F.): We know

$$f_r = \frac{1}{2\pi\sqrt{LC}} \qquad B.W. = \frac{R}{2\pi L} \qquad Q.F. = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$(B.W.) \times (Q.F.) = \frac{R}{2\pi L} \times \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{2\pi\sqrt{LC}} = f_r$$

$$(B.W.) \times (Q.F.) = f_r$$

3.13 PARALLEL RESONANCE

When resonance is create in parallel RLC circuit then it is called parallel resonance. A parallel RLC circuit and its phasor diagram is shown in fig. 3.30 (a) and 3.30 (b) respectively.

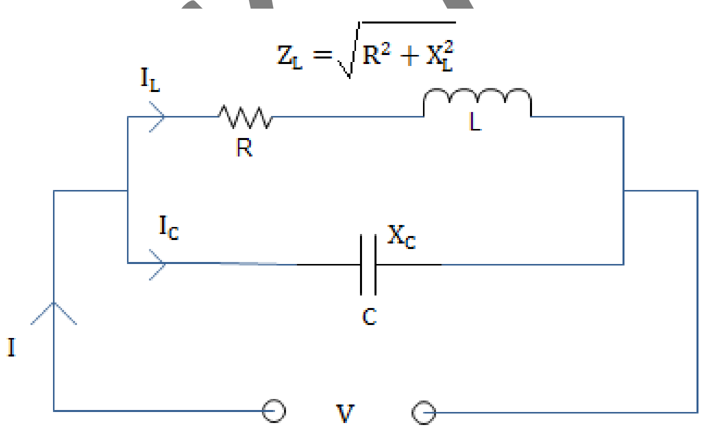


Fig. 3.30 (a)

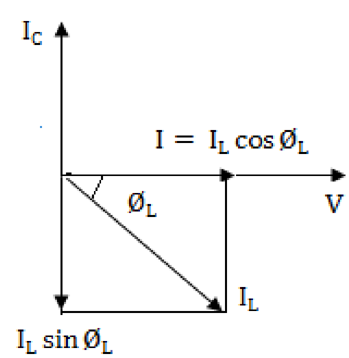


Fig. 3.30 (b)

Conditions for parallel resonance are

(1) $I_C = I_L \sin Ø_L$

(2) Impedance is Maximum
(3) Phase Angle $\emptyset = 0$
(4) Power Factor $\cos\emptyset = 1$
(5) Current is Minimum

Resonant Frequency: We have

$$I_C = I_L \sin\emptyset_L$$

From the circuit diagram shown in fig. 3.30 (a)

$$I_C = \frac{V}{X_C} \qquad I_L = \frac{V}{Z_L} \qquad \sin\emptyset_L = \frac{X_L}{Z_L}$$

So

$$\frac{V}{X_C} = \frac{V}{Z_L}\frac{X_L}{Z_L}$$

$$Z_L^2 = X_L X_C = 2\pi f L \times \frac{1}{2\pi f C} = \frac{L}{C}$$

$$R^2 + X_L^2 = \frac{L}{C}$$

$$X_L^2 = \frac{L}{C} - R^2$$

$$X_L = \sqrt{\frac{L}{C} - R^2}$$

$$2\pi f_r L = \sqrt{\frac{L}{C} - R^2}$$

$$f_r = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If

$$\frac{R}{L} < \frac{1}{LC} \text{ then } \left(\frac{R}{L}\right)^2 << \frac{1}{LC}$$

So resonant frequency

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

The resonant frequency in series and parallel RLC circuit are same if

$$\left(\frac{R}{L}\right)^2 << \frac{1}{LC}$$

Dynamic impedance: Total impedance of the RLC parallel circuit in the condition of resonance is called the dynamic impedance. It is denoted by Z_D.

We know

$$I = I_L \cos\phi_L$$

From the circuit diagram shown in fig. 3.30 (a)

$$I = \frac{V}{Z_D} \qquad I_L = \frac{V}{Z_L} \qquad \cos\phi_L = \frac{R}{Z_L}$$

so

$$\frac{V}{Z_D} = \frac{V}{Z_L}\frac{R}{Z_L}$$

$$Z_D = \frac{Z_L^2}{R}$$

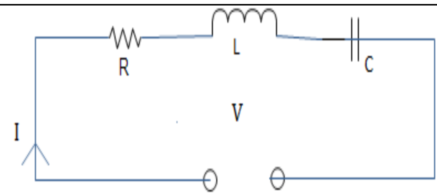
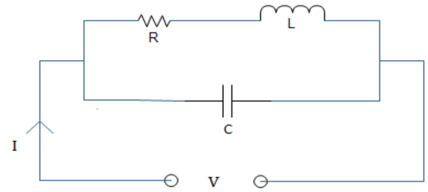
Substitute the value of Z_L^2

$$Z_D = \frac{L}{RC}$$

3.14 COMPARISON BETWEEN SERIES AND PARALLEL RESONANCE

The comparison between series and parallel resonance is in table 3.1

Table 3.1

PARAMETER	SERIES RESONANCE	PARALLEL RESONANCE
Circuit		
Resonant frequency	$f_r = \frac{1}{2\pi\sqrt{LC}}$	$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
Power factor	$\cos\emptyset = 1$	$\cos\emptyset = 1$
Impedance	$Z = R(\text{minimum})$	$Z_D = \frac{L}{RC}(\text{maximum})$
Current	$I = V/R(\text{maximum})$	$I = VRC/L(\text{minimum}$
Bandwidth	$R/2\pi L$	$R/2\pi L$
Quality factor	$\frac{1}{R}\sqrt{\frac{L}{C}}$	$\frac{1}{R}\sqrt{\frac{L}{C}}$
Magnify	Voltage	Current
Application	FM tuning	Filter

Example 3.16 A series RLC circuit has $R = 20\ \Omega, L = 30\ \text{mH}$ and $C = 20\ \mu\text{F}$ Determine (1) Resonant frequency (2) Band width (3) Quality Factor (4) Half power frequency

Solution: (1) Resonant frequency (f_r): We know in series RLC circuit

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{30\times10^{-3}\times20\times10^{-6}}} = 205.468\ \text{Hz}\ \textbf{Ans.}$$

(1) Band width: We know

$$\text{B.W.} = \frac{R}{2\pi L} = \frac{20}{2\times\pi\times30\times10^{-3}} = 106.1\ \text{Hz}\ \textbf{Ans.}$$

(2) Quality Factor: We know

$$\text{Q.F.} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{20}\sqrt{\frac{30\times10^{-3}}{20\times10^{-6}}} = 1.94\ \textbf{Ans.}$$

(3) Half power frequency ($f_{1,}f_2$): We know

$$f_1 = f_r - \frac{R}{4\pi L} = 205.468 - \frac{20}{4\times\pi\times30\times10^{-3}} = 205.468 - 53.05 = 152.418\ \text{Hz}\ \textbf{Ans.}$$

$$f_2 = f_r + \frac{R}{4\pi L} = 205.468 + \frac{20}{4\times\pi\times30\times10^{-3}} = 205.468 + 53.05 = 258.518\ \text{Hz}\ \textbf{Ans.}$$

3.15 POWER FACTOR IMPROVEMENT

The power factor for lightning load ranges from 0.95 to unity but for single phase motor may be as low as 0.4.

Disadvantages of low power factor: for constant power P and voltage V the current I will depend on the power factor.

$$I = \frac{P}{V \cos \varphi}$$

The current is inversely proportional to the power factor $(\cos \varphi)$

If power factor is low current is high and

If power factor is high current is low.

The higher current due to low power factor affects the system and result in the following disadvantages.

(i) More copper material is requires in wire, cable to handle the more current.
(ii) Copper losses will be increase, so efficiency will be decrease.
(iii) Rating of the generators and transformers will increase.
(iv) The cross sectional area of the bus bar is increased.

Causes of low power factor:

(i) All ac motors operate at low power factor.
(ii) Arc lamps operate at low power factor.
(iii) Industrial heating furnaces operate on very low power factor.

Advantage of high power factor: If the power factor is around unity, for same amount of power the value of current is reduced. The reduction of current has the following advantage

(i) No extra copper material is requires in wire, cable to handle the more current.
(ii) Copper losses will be reduced, so efficiency will be increase.
(iii) Rating of the generators and transformers will not increase.

Method to improve the power factor: The power factor can be improved by

(i) Connecting the static capacitor in parallel with the equipment operating at low power factor
(ii) Using induction motor with phase advancers
(iii) Using synchronous condensers like shunt capacitors connected across the supply.