UNIT 1

NETWORK THEOREM

1.1 SUPERPOSITION THEOREM

According to this theorem "The current through any resistance is the algebraic sum of all the current produced by each source individually, when all other sources replaced by there internal resistance."

Limitation of superposition theorem:

(1) Superposition theorem is applicable only on linear circuit.
(2) Superposition theorem is not applicable on the circuit containing dependent source.

Steps to apply superposition theorem

(1) Consider a single source at a time and remove all other sources.
Voltage source short circuited.
Current source open circuited.
(2) Find the current in the desired resistance in the same direction using mesh analysis.
(3) Repeat the step (1) and (2) for all the sources and find the current in the desired resistance from each source.
(4) Find the total current in the desired resistance by adding current from each source in the desired resistance.

Example 4.1 Find the current in 30 Ω resistance using superposition theorem shown in fig. 4.1 (a).

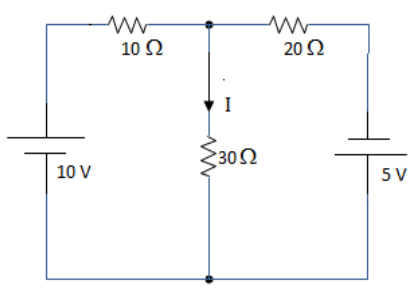


Fig. 4.1 (a)

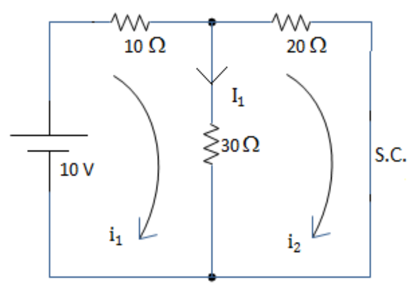


Fig. 4.1 (b)

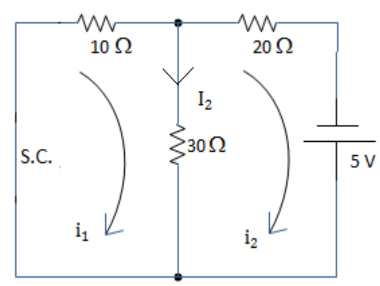


Fig 4.1 (c)

Solution: There are total 2 sources.

Case 1: (1) Consider 10 V voltage source and 5 V voltage source short circuited as shown in fig. 4.1 (b).

(2) The desired resistance is 30 Ω. So assume the current I_1 in the same asking direction. Now apply the mesh analusis in fig. 4.1 (b) and find the value of I_1

In first mesh $\quad 10 - 10i_1 - 30(i_1 - i_2) = 0 \quad \Rightarrow \quad -40i_1 + 30i_2 = -10$ ....(4.1)

In second mesh $30(i_1 - i_2) - 20i_2 = 0 \quad \Rightarrow \quad 30i_1 - 50i_2 = 0$ ....(4.2)

By solving eq. 4.1 and 4.2 $i_1 = 0.45 \text{ Amp.} \quad i_2 = 0.27 \text{ Amp.}$

So in case 1 current in 30 Ω resistance $I_1 = i_1 - i_2 = 0.45 - 0.27 = 0.18 \text{ Amp.}$

Case 2: (1) Consider 5 V voltage source and 10 V voltage source short circuited as shown in fig. 4.1 (c).

(2) The desired resistance is 30 Ω. So assume the current I_2in the same asking direction. Now apply the mesh analysis in fig. 4.1 (c) and find the value of I_2

In first mesh $-10i_1 - 30(i_1 - i_2) = 0 \quad \Rightarrow \quad -40i_1 + 30i_2 = 0$ ....(4.3)

In second mesh $30(i_1 - i_2) - 20i_2 + 5 = 0 \quad \Rightarrow \quad 30i_1 - 50i_2 = -5$ ....(4.4)

By solving eq. 4.3 and 4.4 $i_1 = 0.136 \text{ Amp.} \quad i_2 = 0.182 \text{ Amp.}$

So in case 2 current in 30 Ω resistance $I_2 = i_1 - i_2 = 0.136 - 0.182 = -0.046 \text{ Amp.}$

(3) Completed the step (1) and (2) for each source.

(4) Total current in 30 Ω resistance (I) $I = I_1 + I_2 = 0.18 + (-0.046) = 0.134 \text{ Amp.}$ **Ans.**

Example 4.2 Find the current in 20 Ω resistance using superposition theorem shown in fig. 4.2 (a).

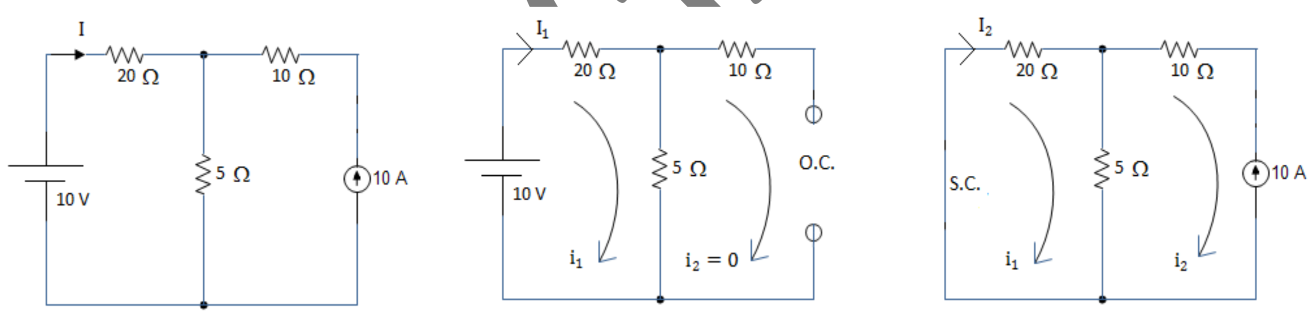


Fig. 4.2 (a) **Fig. 4.2 (b)** **Fig. 4.2 (c)**

Solution: There are total 2 sources.

Case 1: (1) Consider 10 V voltage source and 10 A current source open circuited as shown in fig 4.2 (b).

Step (2) The desired resistance is 20 Ω. So assume the current I_1 in the same asking direction. Now apply the mesh analusis in fig. 4.2 (b) and find the value of I_1.

In first mesh $10 - 20i_1 - 5(i_1 - i_2) = 0 \quad \Rightarrow \quad -25i_1 + 5i_2 = -10$ ....(4.5)

In second mesh $i_2 = 0$ ....(4.6)

By solving eq. 4.5 an 4.6 $i_1 = 0.4 \text{ Amp.} \quad i_2 = 0 \text{ Amp.}$

So in case 1 current in 20 Ω resistance $\quad I_1 = i_1 = 0.4\text{ Amp.}$

Case 2: (1) Consider 10 A current source and 10 V voltage source short circuited as shown in fig. 4.2 (c).

(2) The desired resistance is 20 Ω. So assume the current I_2 in the same asking direction. Now apply the mesh analysis in fig. 4.2 (c) and find the value of I_2

In first mesh $\quad -20i_1 - 5(i_1 - i_2) = 0 \quad \Rightarrow \quad -25i_1 + 5i_2 = 0$ ....(4.7)

In second mesh $\quad i_2 = -10$ ....(4.8)

By solving eq. 4.7 and 4.8 $\quad i_1 = -2\text{ Amp.} \quad i_2 = -10\text{ Amp.}$

So in case 2 current in 20 Ω resistance $\quad I_2 = i_1 = -2\text{ Amp.}$

(3) Completed the step (1) and (2) for each source

(4) Total current in 20 Ω resistance (I) $\quad I = I_1 + I_2 = 0.4 + (-2) = -1.6\text{ Amp. } \textbf{Ans}.$

Example 4.3 Find the current in 10 Ω resistance using superposition theorem shown in fig. 4.3 (a).

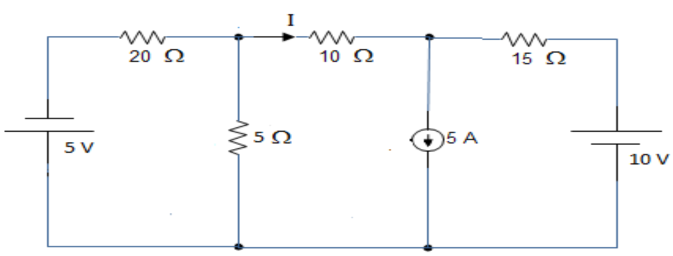


Fig. 4.3 (a)

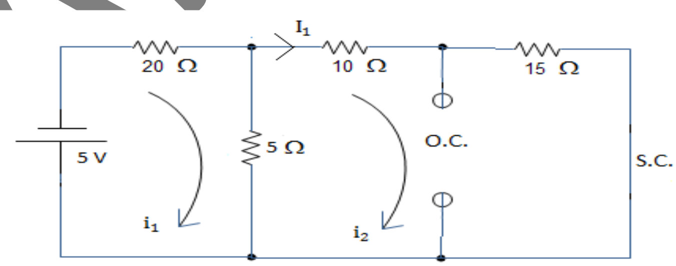


Fig. 4.3 (b)

Solution: There are total 3 sources.

Case 1: (1) Consider 5 V voltage source and 10 V voltage source short circuited and 5 A current source open circuited as shown in fig. 4.3 (b).

(2) The desired resistance is 10 Ω. So assume the current I_1 in the same asking direction. Now apply the mesh analusis in fig. 4.3 (b) and find the value of I_1

In first mesh $\quad -5 - 20i_1 - 5(i_1 - i_2) = 0 \quad \Rightarrow \quad -25i_1 + 5i_2 = 5$ ....(4.9)

In second mesh $\quad 5(i_1 - i_2) - 10i_2 - 15i_2 = 0 \quad \Rightarrow \quad 5i_1 - 30i_2 = 0$ ....(4.10)

By solving eq. 4.9 and 4.10 $\quad i_1 = -0.207\text{ Amp.} \quad i_2 = -0.034\text{ Amp.}$

So in case 1 current in 10 Ω resistance $\quad I_1 = i_2 = -0.034\text{ Amp.}$

Case 2: (1) Consider 5 A current source and 10 V and 5 V voltage source short circuited as shown in fig. 4.4 (a).

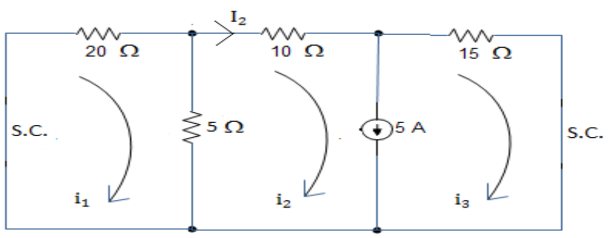


Fig. 4.4 (a)

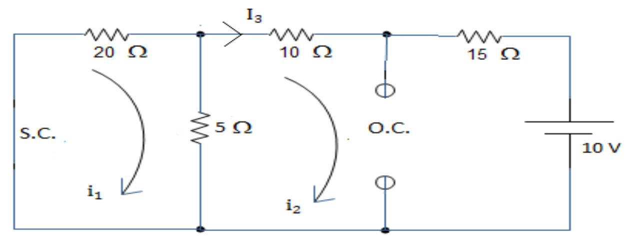


Fig. 4.4 (b)

(2) The desired resistance is 10 Ω. So assume the current I_2 in the same asking direction. Now apply the mesh analysis in fig. 4.4 (a) and find the value of I_2

In first mesh $-20i_1 - 5(i_1 - i_2) = 0 \quad \Rightarrow \quad -25i_1 + 5i_2 = 0$ ....(4.11)

In second or third mesh current source is common so apply the KVL by combined it

$5(i_1 - i_2) - 10i_2 - 15i_3 = 0 \quad \Rightarrow \quad 5i_1 - 15i_2 - 15i_3 = 0$ ....(4.12)

From the branch of the current source $i_2 - i_3 = 5$ ....(4.13)

By solving eq. 4.11, 4.12 and 4.13 $i_1 = 0.517$ Amp. $i_2 = 2.586$ Amp. $i_3 = -2.414$ Amp.

So in case 2 current in 10 Ω resistance $I_2 = i_2 = 2.586$ Amp.

Case 3: (1) Consider 10 V voltage source and 5 V voltage source short circuited and 5 A current source open circuited as shown in fig. 4.4 (b).

(2) The desired resistance is 10 Ω. So assume the current I_3 in the same asking direction. Now apply the mesh analysis in fig. 4.4 (b) and find the value of I_3

In first mesh $-20i_1 - 5(i_1 - i_2) = 0 \quad \Rightarrow \quad -25i_1 + 5i_2 = 0$ ....(4.14)

In second mesh $5(i_1 - i_2) - 10i_2 - 15i_2 - 10 = 0 \quad \Rightarrow \quad 5i_1 - 30i_2 = 10$ ....(4.15)

By solving eq. 4.14 and 4.15 $i_1 = -0.069$ Amp. $i_2 = -0.345$ Amp.

So in case 3 current in 10 Ω resistance $I_3 = i_2 = -0.345$ Amp.

(3) Completed the step (1) and (2) for each source.

(4) Total current in 20 Ω resistance (I) $I = I_1 + I_2 + I_3 = -0.034 + 2.586 - 0.345 = 2.207$ Amp. **Ans.**

Example 4.4 Find the current in all the resistance using superposition theorem shown in fig. 4.5 (a).

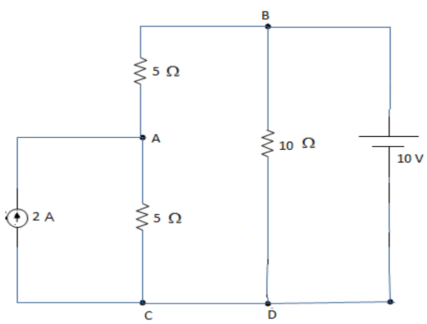


Fig. 4.5 (a)

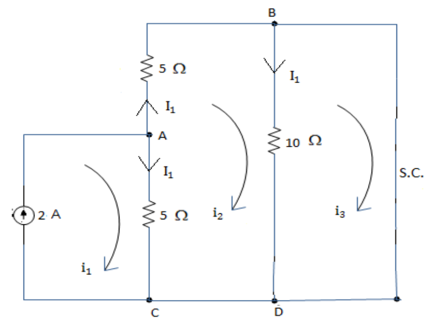


Fig 4.5 (b)

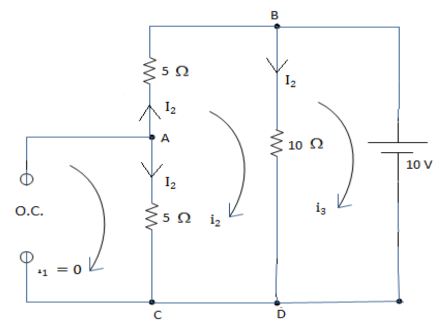


Fig 4.5 (c)

Solution: There are total 2 sources.

Case 1: (1) Consider 2 A current source and 10 V voltage source short circuited as shown in fig. 4.5 (b).

(2) We have to find the current in all the resistance. so assume a current I_1 in all the resistance in any direction. Now apply the mesh analysis in fig. 4.5 (b) and find the value of I_1 in all the resistances.

In first mesh $$i_1 = 2 \qquad ....(4.16)$$

In second mesh $$5(i_1 - i_2) - 5i_2 - 10(i_2 - i_3) = 0 \quad \Rightarrow \quad 5i_1 - 20i_2 + 10i_3 = 0 \qquad ....(4.17)$$

In third mesh $$10(i_2 - i_3) = 0 \quad \Rightarrow \quad 10i_2 - 10i_3 = 0 \qquad ....(4.18)$$

By solving eq. 4.16, 4.17 and 4.18 $\quad i_1 = 2, i_2 = 1, i_3 = 1$

$$\text{In case 1 current in 5 } \Omega \text{ (lower) resistance } I_{1(5\,\Omega\,\text{lower})} = i_1 - i_2 = 2 - 1 = 1 \text{ Amp.}$$

$$\text{In case 1 current in 5 } \Omega \text{ (upper) resistance } I_{1(5\,\Omega\,\text{upper})} = i_2 = 1 \text{ Amp.}$$

$$\text{In case 1 current in 10 } \Omega \text{ resistance } I_{1(10\,\Omega)} = i_2 - i_3 = 1 - 1 = 0 \text{ Amp.}$$

Case 2: (1) Consider 10 V voltage source and 2 A current source open circuited as shown in fig. 4.5 (c).

(2) We have to find the current in all the resistance.so assume a current I_2 in all the resistance in same direction of the case 1. Now apply the mesh analysis in fig. 4.5 (c) and find the value of I_2 in all the resistances.

In first mesh $$i_1 = 0 \qquad ....(4.19)$$

In second mesh $$5(i_1 - i_2) - 5i_2 - 10(i_2 - i_3) = 0 \quad \Rightarrow \quad 5i_1 - 20i_2 + 10i_3 = 0 \qquad ....(4.20)$$

In third mesh $$10(i_2 - i_3) - 10 = 0 \quad \Rightarrow \quad 10i_2 - 10i_3 = 10 \qquad ....(4.21)$$

By solving eq. 4.19, 4.20 and 4.21 $\quad i_1 = 0, i_2 = -1, i_3 = -2$

$$\text{In case 2 current in 5 } \Omega \text{ (lower) resistance } I_{2(5\,\Omega\,\text{lower})} = i_1 - i_2 = 0 + 1 = 1 \text{ Amp.}$$

$$\text{In case 2 current in } 5\ \Omega \text{ (upper) resistance } I_{2(5\,\Omega\,\text{upper})} = i_2 = -1 \text{ Amp.}$$

$$\text{In case 2 current in } 10\ \Omega \text{ resistance } I_{2(10\,\Omega)} = i_2 - i_3 = -1 + 2 = 1 \text{ Amp}$$

(3) Completed the step (1) and (2) for each source

(4) Total current in all the resistance

$$\text{Total current in } 5\ \Omega \text{ (lower) resistance} = I_{1(5\,\Omega\,\text{lower})} + I_{2(5\,\Omega\,\text{lower})} = 1 + 1 = 2 \text{ Amp. } \mathbf{Ans}.$$

$$\text{Total current in } 5\ \Omega \text{ (upper) resistance} = I_{1(5\,\Omega\,\text{upper})} + I_{2(5\,\Omega\,\text{upper})} = 1 - 1 = 0 \text{ Amp. } \mathbf{Ans}.$$

$$\text{Total current in } 10\ \Omega \text{ resistance} = I_{1(10\,\Omega)} + I_{2(10\,\Omega)} = 0 + 1 = 1 \text{ Amp. } \mathbf{Ans}.$$

1.2 NORTON THEOREM

According to this theorem "Any linear bilateral active network connected across any two terminals can be converted in to a current source".

In this current source a Norton equivalent current (I_N) are connected in parallel with the Norton equivalent resistance (R_N) as shown in fig. 4.6 (b). The circuit shown in fig. 4.6 (b) is called the Norton Equivalent Circuit (NEC).

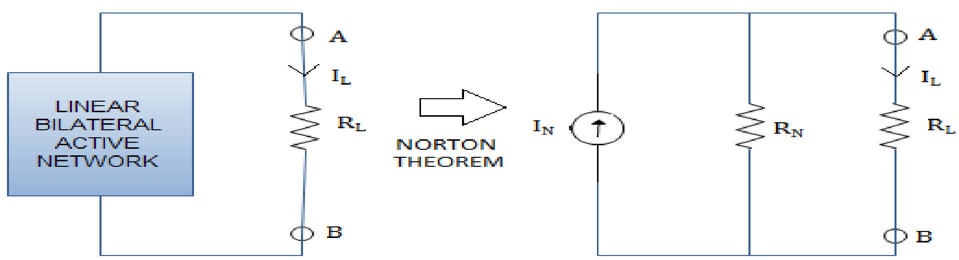


Fig. 4.6 (a) **Fig. 4.6 (b)**

From the circuit shown in fig. 4.6 (b)

$$I_L = \frac{I_N R_N}{R_N + R_L}$$

Where R_L is the load resistance. It is the resistance in which we have to find the current.

Steps to find the current using Norton theorem

(1) Find the resistance in which we have to find the current. It is your load resistance (R_L). Mark the terminal A-B across the load resistance.
(2) Find I_N: For that apply the following three steps.
    (i) Remove the load resistance R_L and make the terminal A and B short circuited.
    (ii) Find the current in all the meshes using mesh analysis.
    (iii) Find the current between terminal A and B ($I_{AB} = I_N$).

(3) Find R_N:For that apply the following three steps
   (i) Remove the load resistance R_L and make the terminal A and B open circuited.
   (ii) Make the all voltage source short circuited and all current source open circuited.
   (iii) Find the resistance between terminal A and B. For that simplify the circuit and see the series parallel combination of the resistances between terminals A-B. The value of resistance between terminals A-B is R_N.
(4) Draw the Norton Equivalent Circuit (NEC).
(5) Find the current in the load resistance using

$$I_L = \frac{I_N R_N}{R_N + R_L}$$

Example 4.5 Find the current in 30 Ω resistance using Norton theorem in the circuit shown in fig. 4.7 (a).

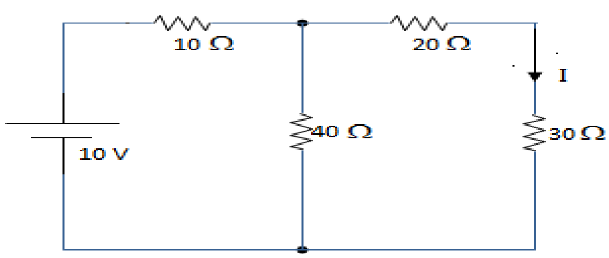


Fig. 4.7 (a)

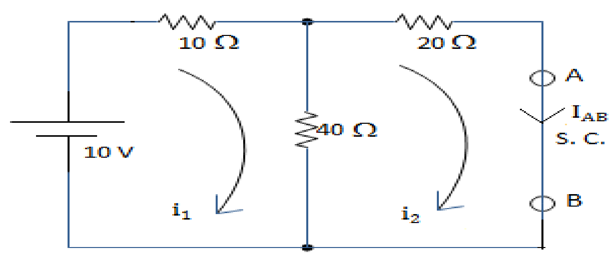


Fig 4.7 (b)

Solution:

(1) We have to find the current in 30 Ω resistance, So 30 Ω is our load resistance R_L. Mark the terminal A-B across the R_L.

(2) Find I_N: For that apply the following three steps.

(i) Remove the load resistance R_L and make the terminal A and B short circuited as shown in fig. 4.7 (b).

(ii) Find the current in all the meshes using mesh analysis.

In first mesh $10 - 10i_1 - 40(i_1 - i_2) = 0 \quad \Rightarrow \quad -50i_1 + 40i_2 = -10$ ....(4.22)

In second mesh $40(i_1 - i_2) - 20i_2 = 0 \quad \Rightarrow \quad 40i_1 - 60i_2 = 0$ ....(4.23)

By solving eq. 4.22 and 4.23 $\quad i_1 = 0.428 \text{ Amp.} \quad i_2 = 0.286 \text{ Amp.}$

(iii) Find the current between terminal A and B . $\quad I_{AB} = I_N = i_2 = 0.286 \text{ Amp.}$

(3) Find R_N: For that apply the following three steps.

(i) Remove the load resistance R_L and make the terminal A and B open circuited as shown in fig. 4.8 (a).

(ii) Make the all voltage source short circuited and all current source open circuited as shown in fig. 4.8 (a).

(iii) Now draw the simplify circuit as shown in fig. 4.8 (b), from this circuit it is clear that 10 Ω and 40 Ω are in parallel and the resultant is in series with the 20 Ω resistance.

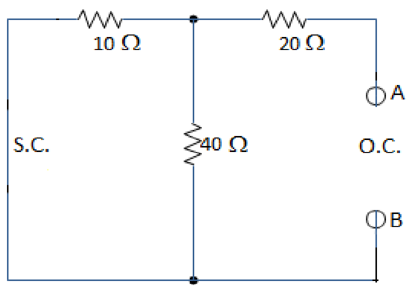


Fig. 4.8 (a)

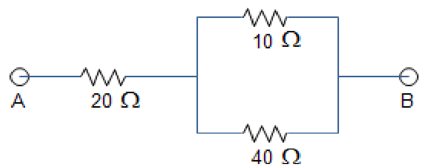


Fig. 4.8 (b)

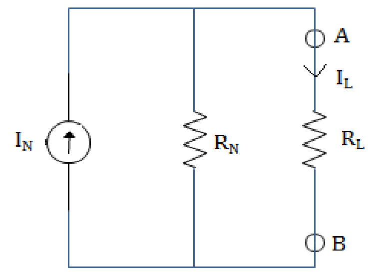


Fig. 4.8 (c)

$$R_{AB} = R_N = 20 + \frac{10 \times 40}{10 + 40} = 20 + 8 = 28\ \Omega$$

Step (4) Draw the Norton Equivalent Circuit(NEC): The Norton equivalent circuit is shown in fig. 4.8 (c).

Step (5) Find the current in the load resistance using

$$I_L = \frac{I_N R_N}{R_N + R_L} = \frac{0.286 \times 28}{28 + 30} = 0.138\ \text{Amp.} \quad \textbf{Ans.}$$

Example 4.6 Find current in the 30 Ω resistance using Norton theorem for circuit shown in fig. 4.9 (a).

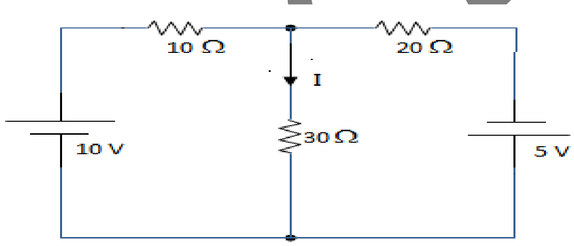


Fig. 4.9 (a)

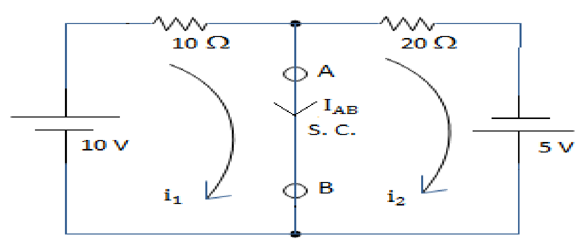


Fig. 4.9 (b)

Solution:

(1) We have to find the current in 30 Ω resistance. So 30 Ω is our load resistanceR_L. Mark the terminal A-B across the R_L .

(2) Find I_N: For that apply the following three steps.

(i) Remove the load resistance R_L and make the terminal A and B short circuited as shown in fig. 4.9 (b).

(ii) Find the current in all the meshes using mesh analysis.

In first mesh $10 - 10i_1 = 0 \Rightarrow 10i_1 = 10 \Rightarrow i_1 = 1$ ....(4.24)

In second mesh $-20i_2 + 5 = 0 \Rightarrow 20i_2 = 5 \Rightarrow i_2 = 0.25$ ....(4.25)

By solving eq. 4.24 and 4.25 $i_1 = 1\ \text{Amp.},\ \ i_2 = 0.25\ \text{Amp.}$

(iii) Find the current between terminal A and B. $I_{AB} = I_N = i_1 - i_2 = 1 - 0.25 = 0.75\ \text{Amp.}$

(3) Find R_N: For that apply the following three steps

(i) Remove the load resistance R_L and make terminal A and B open circuited as shown in fig. 4.10 (a).

(ii) Make all voltage source short circuited and all current source open circuited as shown in fig. 4.10 (a).

(iii) Now draw the simplify circuit as shown in fig. 4.10 (b), from this circuit it is clear that 10 Ω and 40 Ω are in parallel.

$$R_{AB} = R_N = \frac{10 \times 40}{10 + 40} = 8\ \Omega$$

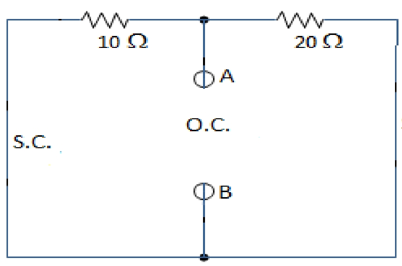


Fig. 4.10 (a)

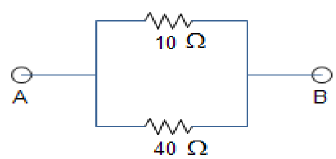


Fig. 4.10 (b)

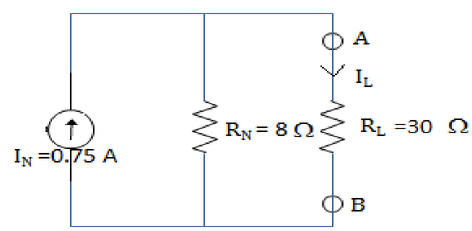


Fig. 4.10 (c)

(4) Draw the Norton Equivalent Circuit (NEC): The Norton equivalent circuit is shown in fig. 4.10 (c).

(5) Find the current in the load resistance using

$$I_L = \frac{I_N R_N}{R_N + R_L} = \frac{0.75 \times 8}{8 + 30} = \frac{6}{38} = 0.158\ \text{Amp.}\ \ \textbf{Ans.}$$

Example 4.7 Find current in the 30 Ω resistance using Norton theorem for circuit shown in fig. 4.11 (a).

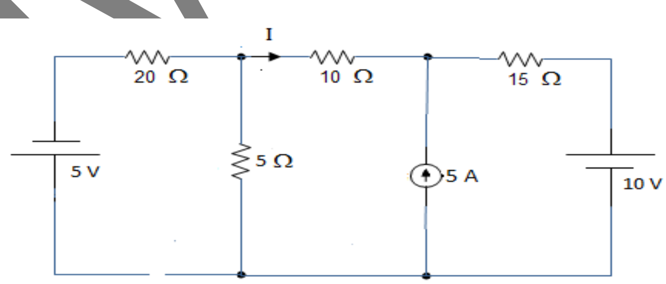


Fig. 4.11 (a)

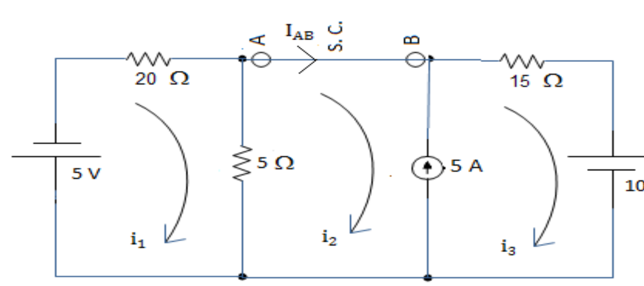


Fig. 4.11 (b)

Solution:

(1) We have to find the current in 10 Ω resistance. So 10 Ω is our load resistanceR_L. Mark the terminal A-B across the R_L .

(2) Find I_N: For that apply the following three steps.

(i) Remove load resistance R_L and make the terminal A and B short circuited as shown in fig. 4.11 (b).

(ii) Find the current in all the meshes using mesh analysis.

In first mesh $\quad -5-20i_1-5(i_1-i_2)=0 \quad \Rightarrow \quad -25i_1+5i_2=5$ ....(4.26)

In second or third mesh current source is common so apply the KVL by combined it

$$5(i_1-i_2)-15i_3-10=0 \quad \Rightarrow \quad 5i_1-5i_2-15i_3=10 \qquad ....(4.27)$$

From the branch of the current source $\quad i_2-i_3=-5$ ....(4.28)

By solving eq. 4.26, 4.27 and 4.28 $\quad i_1=-1.1\text{ Amp.},\; i_2=-4.53\text{ Amp.},\; i_2=0.47\text{ Amp.}$

(iii) Find the current between terminal A and B. $\quad I_{AB}=I_N=i_2=-4.53\text{ Amp.}$

(3) Find R_N: For that apply the following three steps

(i) Remove load resistance R_L and make the terminal A and B open circuited as shown in fig. 4.12 (a).

(ii) Make all voltage source short circuited and all current source open circuited as shown in fig. 4.12 (a).

(iii) Now draw the simplify circuit as shown in fig. 4.12 (b) from this circuit it is clear that 20 Ω and 5 Ω are in parallel and the resultant is in series with 15 Ω resistance.

$$R_{AB}=R_N=15+\frac{20\times 5}{20+5}=15+4=19\,\Omega$$

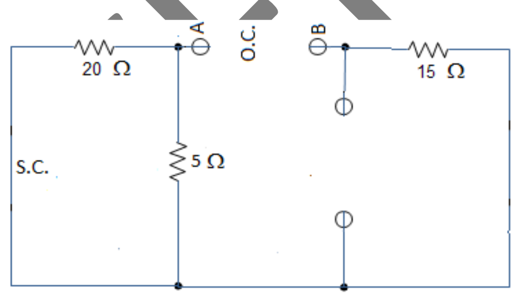


Fig. 4.12 (a)

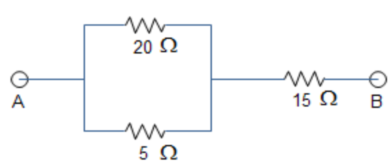


Fig. 4.12 (b)

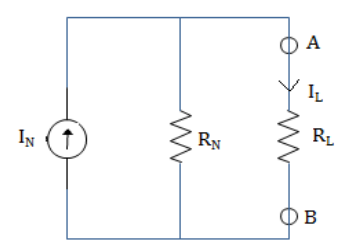


Fig. 4.13 (c)

(4) Draw the Norton Equivalent Circuit (NEC): The Norton equivalent circuit is shown in fig. 4.13 (c).

(5) Find the current in the load resistance using

$$I_L = \frac{I_N R_N}{R_N + R_L} = \frac{(-4.53) \times 19}{19 + 10} = \frac{-86.07}{29} = -2.97 \text{ Amp. } \textbf{Ans.}$$

Example 4.8 Find current in the 15 Ω resistance using Norton theorem for circuit shown in fig. 4.13 (a).

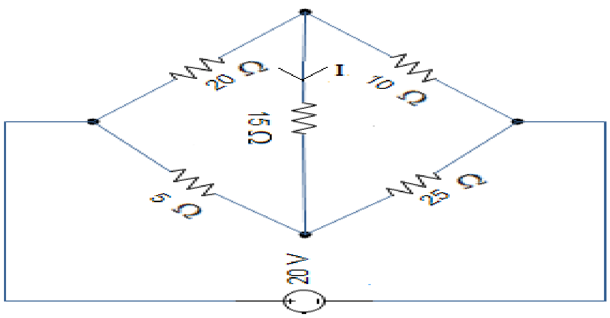


Fig. 4.13 (a)

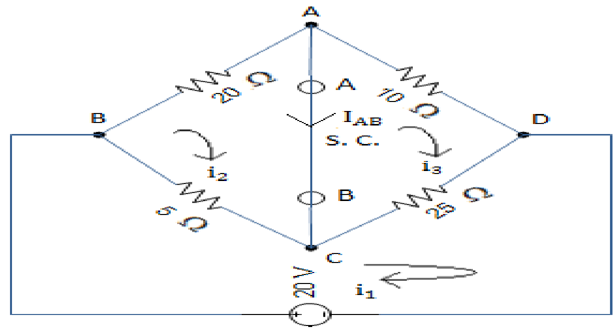


Fig. 4.13 (b)

Solution:

(1) We have to find the current in 15 Ω resistance. So 15 Ω is our load resistance R_L. Mark the terminal A-B across the R_L .

(2) Find I_N: For that apply the following three steps.

(i) Remove load resistance R_L and make the terminal A and B short circuited as shown in fig. 4.13 (b).

(ii) Find the current in all the meshes using mesh analysis.

In first mesh $\quad 20 - 5(i_1 - i_2) - 25(i_1 - i_3) = 0 \quad \Rightarrow \quad -30i_1 + 5i_2 + 25i_3 = -20$ ....(4.29)

In second mesh $\quad 5(i_1 - i_2) - 20i_2 = 0 \quad \Rightarrow \quad 5i_1 - 25i_2 = 0$ ....(4.30)

In third mesh $\quad 25(i_1 - i_3) - 10i_3 = 0 \quad \Rightarrow \quad 25i_1 - 35i_3 = 0$ ....(4.31)

By solving the eq. 4.29, 4.30 and 4.31 $\quad i_1 = 1.795 \text{ Amp.} \quad i_2 = 0.359 \text{ Amp.} \quad i_3 = 1.282 \text{ Amp.}$

(iii) Find the current between terminal A and B. $\quad I_{AB} = I_N = i_2 - i_3 = 0.359 - 1.282 = -0.923 \text{ Amp.}$

(3) Find R_N: For that apply the following three steps

(i) Remove load resistance R_L and make the terminal A and B open circuited as shown in fig. 4.14 (a).

(ii) Make all voltage source short circuited and all current source open circuited as shown in fig. 4.14 (a).

(iii) Now draw the simplify circuit as shown in fig. 4.14 (b) from this circuit it is clear that 20 Ω and 10 Ω are in parallel and 5 Ω and 25 Ω are in parallel. The resultant of both parallel combination are in series.

$$R_{AB} = R_N = \frac{20 \times 10}{20 + 10} + \frac{5 \times 25}{5 + 25} = 6.67 + 4.17 = 10.84 \ \Omega$$

(4) Draw the Norton Equivalent Circuit (NEC): The Norton equivalent circuit is shown in fig. 4.14 (c).

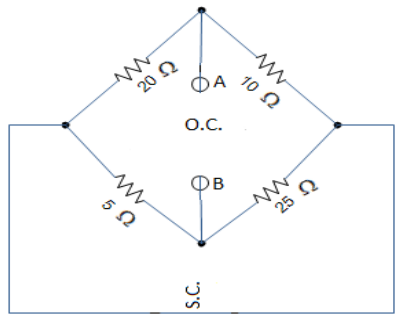


Fig. 4.14 (a)

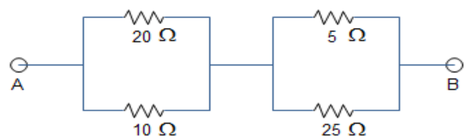


Fig. 4.14 (b)

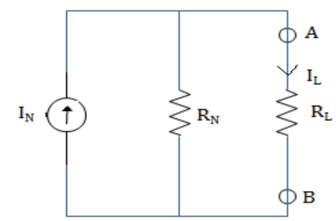


Fig. 4.14 (c)

(5) Find the current in the load resistance using

$$I_L = \frac{I_N R_N}{R_N + R_L} = \frac{(-0.923) \times 10.84}{10.84 + 15} = \frac{-10}{25.84} = -0.387 \text{ Amp. } \textbf{Ans.}$$

Example 4.9 Find Norton equivalent circuit between the terminal A and B shown in fig.4.15 (a).

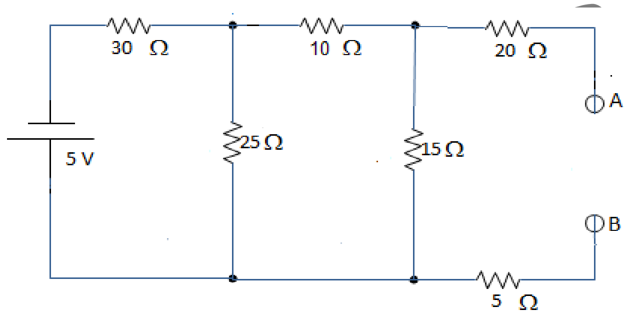


Fig. 4.15 (a)

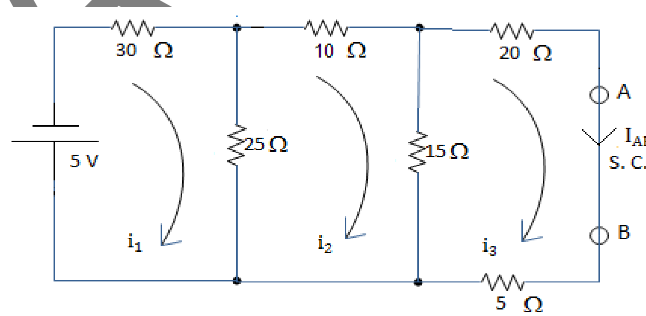


Fig. 4.15 (b)

Solution:

(1) In this problem load resistance is not given. We only have to find I_N and R_N between A and B.

(2) Find I_N: For that apply the following three steps.

(i) Make the terminal A and B short circuited as shown in fig. 4.15 (b).

(ii) Find the current in all the meshes using mesh analysis.

In first mesh $\quad -5 - 30i_1 - 25(i_1 - i_2) = 0 \quad \Rightarrow \quad -55i_1 + 25i_2 = 5 \quad$ ....(4.32)

In second mesh $\quad 25(i_1 - i_2) - 10i_2 - 15(i_2 - i_3) = 0 \quad \Rightarrow \quad 25i_1 - 50i_2 + 15i_3 = 0 \quad$ ....(4.33)

In third mesh $\quad 15(i_2 - i_3) - 20i_3 - 5i_3 = 0 \quad \Rightarrow \quad 15i_2 - 40i_3 = 0 \quad$ ....(4.34)

By solving the eq. 4.32, 4.33 and 4.34 $i_1 = -0.12$ Amp. $i_2 = -0.07$ Amp. $i_3 = -0.026$ Amp.

(iii) Find the current between terminal A and B. $I_{AB} = I_N = i_3 = -0.026$ Amp. **Ans.**

(3) Find R_N: For that apply the following three steps

(i) The terminal A and B open circuited as shown in fig. 4.16 (a).

(ii) Make all voltage source short circuited and all current source open circuited as shown in fig. 4.16 (a).

(iii) Now draw the simplify circuit as shown in fig. 4.16 (b), By solving this circuit.

$$R_{AB} = R_N = 20 + \frac{15\left(10 + \frac{30 \times 25}{30 + 25}\right)}{15 + \left(10 + \frac{30 \times 25}{30 + 25}\right)} + 5 = 20 + 9.18 + 5 = 34.18\,\Omega \ \textbf{Ans.}$$

(4) Draw the Norton Equivalent Circuit (NEC): The Norton equivalent circuit is shown in fig. 4.16 (c). **Ans.**

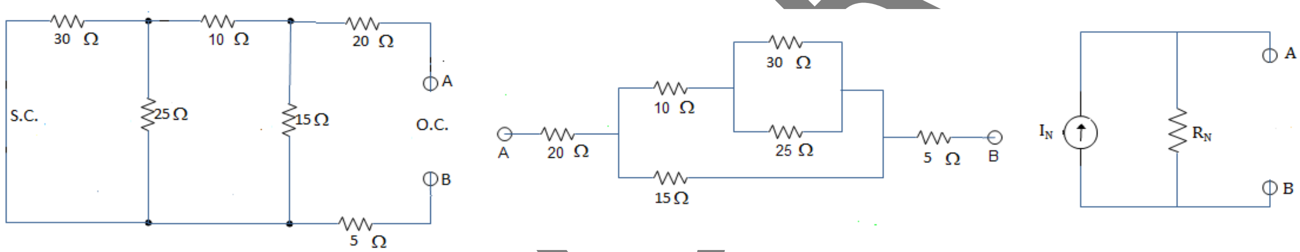


Fig. 4.16 (a) **Fig. 4.16 (b)** **Fig. 4.16 (c)**

4.3 THEVENIN THEOREM

According to this theorem "Any linear bilateral active network connected across any two terminals can be converted in to a voltage source".

In this voltage source a Thevenin equivalent voltage (V_{Th}) are connected in series with the Thevenin equivalent resistance (R_{Th}) as shown in fig. 4.17 (b). The circuit shown in fig. 4.17 (b) is called the Thevenin Equivalent Circuit(TEC)

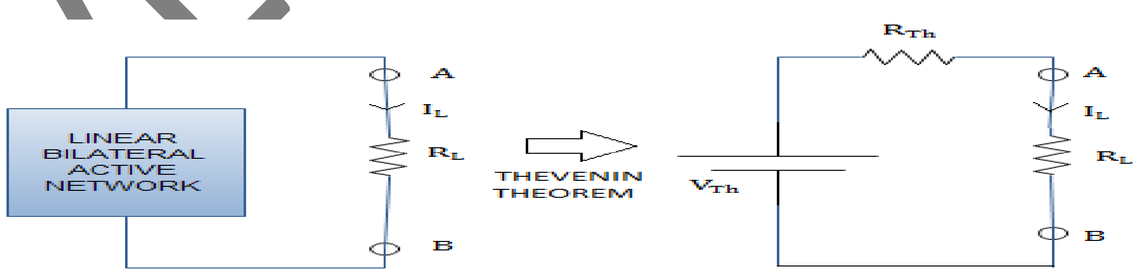


Fig. 4.17 (a) **Fig. 4.17 (b)**

From the circuit shown in fig. 4.17 (b)

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Where R_L is the load resistance. It is the resistance in which we have to find the current.

Steps to find the current using Thevenin theorem.

(1) Find the resistance in which we have to find the current. It is your load resistance (R_L). Mark the terminal A-B across the load resistance.
(2) Find I_N:
(3) Find R_N:
(4) Find V_{Th} : $V_{Th} = I_N R_N$
(5) Find R_{Th}: $R_{Th} = R_N$
(6) Draw the Thevenin Equivalent Circuit (TEC).
(7) Find the current in the load resistance using

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Example 4.10 Find current in the 15 Ω resistance using Thevenin theorem in circuit shown in fig. 4.18 (a).

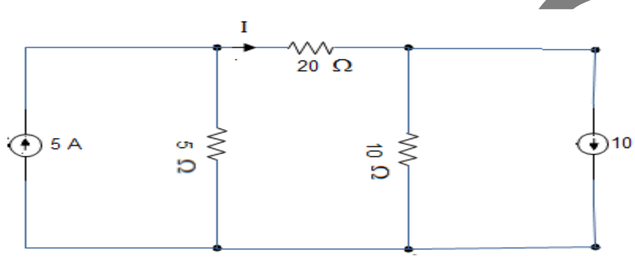


Fig. 4.18 (a)

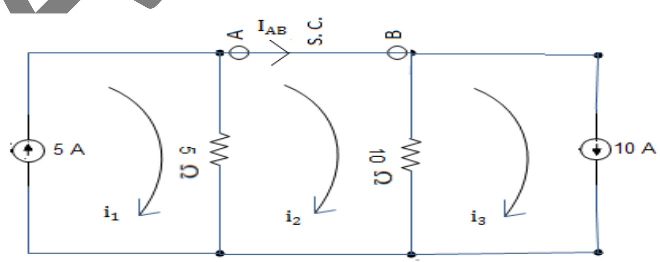


Fig. 4.18 (b)

Solution:

(1) We have to find the current in 20 Ω resistance. So 20 Ω is our load resistanceR_L. Mark the terminal A-B across the R_L .

(2) Find I_N: For that apply the following three steps.

(i) Remove load resistance R_L and make the terminal A and B short circuited as shown in fig. 4.18 (b).

(ii) Find the current in all the meshes using mesh analysis.

In first mesh $i_1 = 5$ ....(4.35)

In second mesh $5(i_1 - i_2) - 10(i_2 - i_3) = 0 \Rightarrow 5i_1 - 15i_2 + 10i_3 = 0$ ....(4.36)

In third mesh $i_3 = 10$ ....(4.37)

By solving the eq. 4.35, 4.36 and 4.37 $i_1 = 5 \text{ Amp.} \quad i_2 = 8.33 \text{ Amp.} \quad i_3 = 10 \text{ Amp.}$

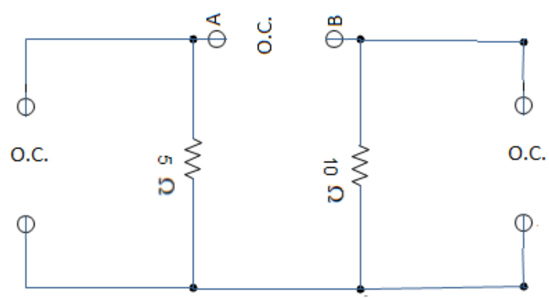
Find the current between terminal A and B . $I_{AB} = I_N = i_2 = 8.33 \text{ Amp.}$

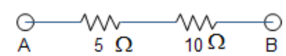
(3) Find R_N: For that apply the following three steps

(i) Remove load resistance R_L and make the terminal A and B open circuited as shown in fig. 4.19 (a).

(ii) Make all voltage source short circuited and all current source open circuited as shown in fig. 4.19 (a).

(iii) Now draw simplify circuit as shown in fig. 4.19 (b) from this circuit it is clear that 5 Ω and 10 Ω are in series.





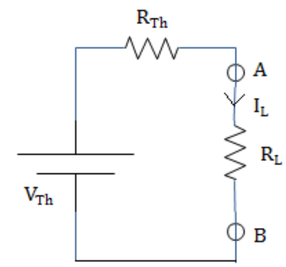


Fig. 4.19 (a) **Fig. 4.19 (b)** **Fig. 4.19 (c)**

$$R_{AB} = R_N = 5 + 10 = 15\ \Omega$$

(4) Find V_{Th}: $V_{Th} = I_N R_N = 8.33 \times 15 = 124.5 \text{ V}$

(5) Find R_{Th}: $R_{Th} = R_N = 15\ \Omega$

(6) The Thevenin equivalent circuit is shown in fig. 4.19 (c).

(7) Find the current in the load resistance using

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{124.5}{15 + 20} = \frac{124.5}{35} = 3.55 \text{ Amp. } \textbf{Ans.}$$

Example 4.11 Find current in the 15 Ω resistance using Thevenin theorem in circuit shown in fig. 4.20 (a).

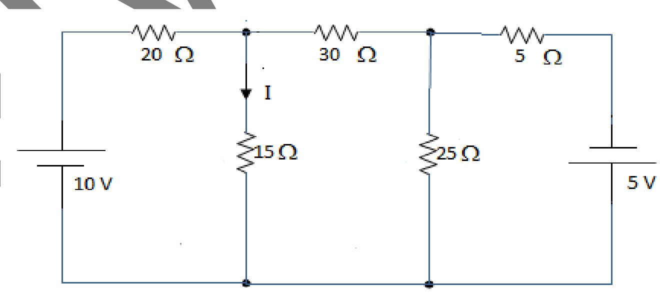


Fig. 4.20 (a)

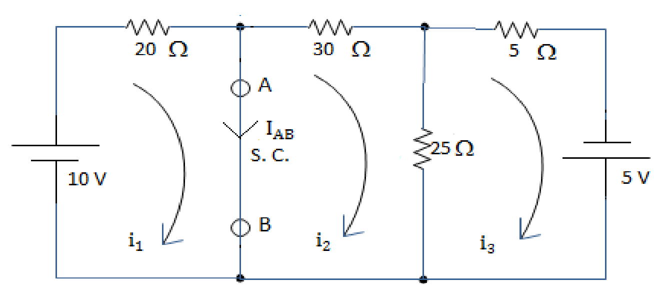


Fig. 4.20 (b)

Solution:

(1) We have to find the current in 15 Ω resistance. So 15 Ω is our load resistance R_L. Mark the terminal A-B across the R_L .

(2) Find I_N: For that apply the following three steps.

(i) Remove load resistance R_L and make the terminal A and B short circuited as shown in fig. 4.20 (b).

(ii) Find the current in all the meshes using mesh analysis.

In first mesh $\quad 10 - 20i_1 = 0 \quad \Rightarrow \quad 20i_1 = 10 \quad \Rightarrow \quad i_1 = 0.5$ ....(4.38)

In second mesh $\quad -30i_2 - 25(i_2 - i_3) = 0 \quad \Rightarrow \quad -55i_2 + 25i_3 = 0$ ....(4.39)

In third mesh $\quad 25(i_2 - i_3) - 5i_3 + 5 = 0 \quad \Rightarrow \quad 25i_2 - 30i_3 = -5$ ....(4.40)

By solving the eq. 4.38, 4.39 and 4.40 $\quad i_1 = 0.5 \text{ Amp.} \quad i_2 = 0.122 \text{ Amp.} \quad i_3 = 0.268 \text{ Amp.}$

Find the current between terminal A and B . $\quad I_{AB} = I_N = i_1 - i_2 = 0.5 - 0.122 = 0.378 \text{ Amp.}$

(3) Find R_N: For that apply the following three steps

(i) Remove load resistance R_L and make the terminal A and B open circuited as shown in fig. 4.21 (a).

(ii) Make all voltage source short circuited and all current source open circuited as shown in fig. 4.21 (a).

(iii) Now draw simplify circuit as shown in fig. 4.21 (b), By solving the circuit.

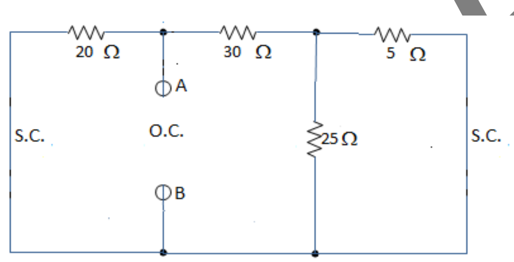


Fig. 4.21 (a)

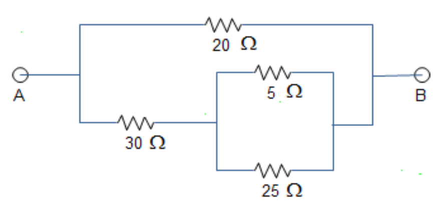


Fig. 4.21 (b)

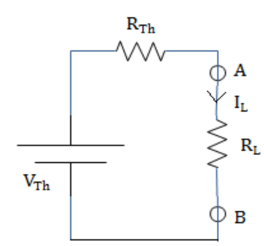


Fig. 4.21 (c)

$$R_{AB} = R_N = \frac{20\left(30 + \frac{5 \times 25}{5 + 25}\right)}{20 + \left(30 + \frac{5 \times 25}{5 + 25}\right)} = 12.61 \ \Omega$$

(4) Find V_{Th}: $\quad V_{Th} = I_N R_N = 0.378 \times 12.61 = 4.77 \text{ V}$

(5) Find R_{Th}: $\quad R_{Th} = R_N = 12.61 \ \Omega$

(6) The Thevenin equivalent circuit is shown in fig. 4.21 (c).

(7) Find the current in the load resistance using

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{4.77}{12.61 + 15} = \frac{4.77}{27.61} = 0.172 \text{ Amp. } \textbf{Ans.}$$

Example 4.12 Find Thevenin equivalent circuit between the terminal A and B shown in fig. 4.22 (a).

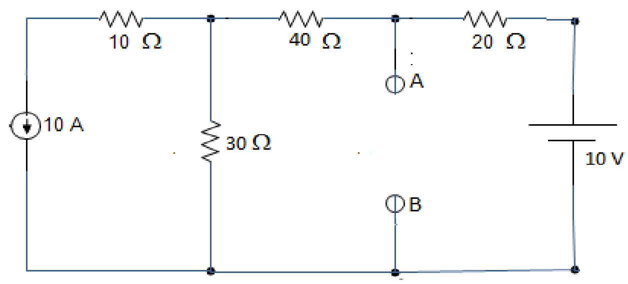


Fig. 4.22 (a)

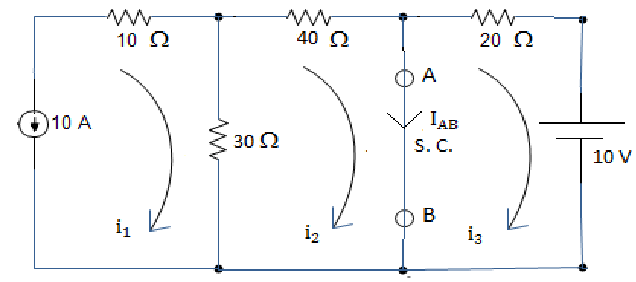


Fig. 4.22 (b)

Solution:

(1) In this problem load resistance is not given. We only have to find V_{Th} and R_{Th} between A and B.

(2) Find I_N: For that apply the following three steps.

(i) Make the terminal A and B short circuited as shown in fig. 4.22 (b).

(ii) Find the current in all the meshes using mesh analysis.

In first mesh $\quad i_1 = -10 \quad$ ....(4.41)

In second mesh $\quad 30(i_1 - i_2) - 40i_2 = 0 \quad \Rightarrow \quad 30i_1 - 70i_2 = 0 \quad$ ....(4.42)

In third mesh $\quad -20i_3 - 10 = 0 \quad \Rightarrow \quad -20i_3 = 10 \quad \Rightarrow \quad i_3 = -0.5 \quad$ ....(4.43)

By solving the eq. 4.41, 4.42 and 4.43 $\quad i_1 = -10$ Amp. $\; i_2 = -4.286$ Amp. $\; i_3 = -0.5$ Amp.

(iii) Find the current between terminal A and B. $I_{AB} = I_N = i_2 - i_3 = -4.286 - (-0.5) = -3.786$ Amp.

(3) Find R_N: For that apply the following three steps

(i) The terminal A and B open circuited as shown in fig. 4.23 (a).

(ii) Make all voltage source short circuited and all current source open circuited as shown in fig. 4.23 (a).

(iii) Now draw the simplify circuit as shown in fig. 4.23 (b), By solving this circuit.

$$R_{AB} = R_N = \frac{20 \times (40 + 30)}{20 + 40 + 30} = 15.56 \ \Omega$$

(4) Find V_{Th}: $\quad V_{Th} = I_N R_N = -3.786 \times 15.56 = -58.91 \text{ V } \textbf{Ans.}$

(5) Find R_{Th}: $R_{Th} = R_N = 15.56\,\Omega$ **Ans.**

(6) The Thevenin equivalent circuit is shown in fig. 4.23 (c). **Ans.**

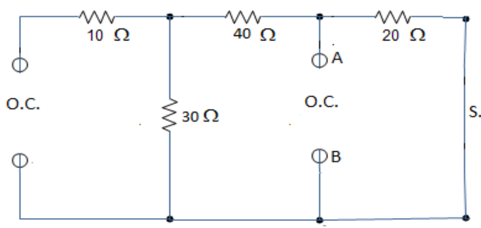


Fig. 4.23 (a)

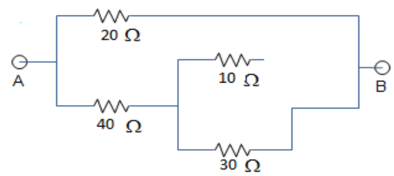


Fig. 4.23 (b)

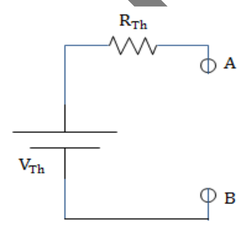


Fig. 4.23 (c)

4.4 MAXIMUM POWER TRANSFER THEOREM

According to this theorem "Maximum power is transfer to the load resistance when load resistance (R_L) is equal to the total internal resistance (R_{Th}) of the circuit".

$$R_L = R_{Th}$$

Proof of maximum power transfer theorem: According to Thevenin Theorem

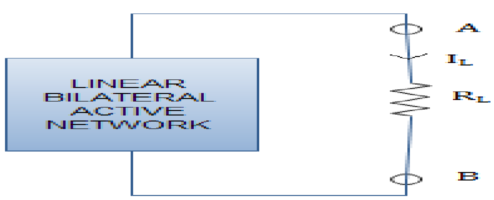


Fig. 4.24 (a)

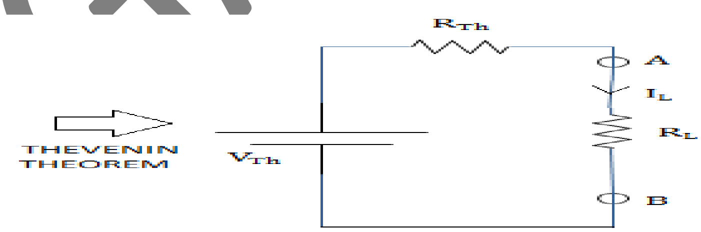


Fig. 4.24 (b)

From the Thevenin Equivalent Circuit (TEC) shown in fig. 4.24 (b) The power in the load resistance

$$P_L = I_L^2 R_L$$

We know

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Substitute the value of I_L . Then the power in the load resistance

$$P_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L = \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2}$$

For the power to be maximum differentiate the power w.r.t load resistance and make it equal to zero

$$\frac{dP_L}{dR_L} = 0$$

$$\Rightarrow \frac{(R_{Th} + R_L)^2 V_{Th}^2 - V_{Th}^2 R_L 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} = 0$$

$$\Rightarrow (R_{Th} + R_L)^2 V_{Th}^2 - V_{Th}^2 R_L 2(R_{Th} + R_L) = 0$$

$$\Rightarrow (R_{Th} + R_L) - 2R_L = 0$$

$$\Rightarrow R_{Th} - R_L = 0$$

$$R_L = R_{Th}$$

The value of maximum power

$$P_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L = \frac{V_{Th}^2 R_{Th}}{(R_{Th} + R_{Th})^2} = \frac{V_{Th}^2 R_{Th}}{(2R_{Th})^2} = \frac{V_{Th}^2 R_{Th}}{4{R_{Th}}^2}$$

$$P_{Lmax} = \frac{V_{Th}^2}{4R_{Th}}$$

Steps to solve the circuit using maximum power transfer theorem.

(1) Find the load resistance R_L and mark the terminal A and B across the load resistance R_L.
(2) Find I_N:
(3) Find R_N:
(4) Find V_{Th} : $V_{Th} = I_N R_N$
(5) Find R_{Th}: $R_{Th} = R_N$
(6) The value of load resistance $R_L = R_{Th}$
(7) The value of maximum power

$$P_{Lmax} = \frac{V_{Th}^2}{4R_{Th}}$$

Example 4.13 In the circuit shown in fig. 4.25 (a), Find the value of load resistance R_L if maximum power will transfer to the load resistance. Also find the value of the maximum power.

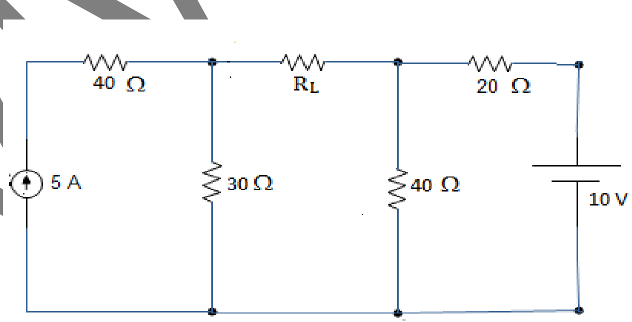


Fig. 4.25 (a)

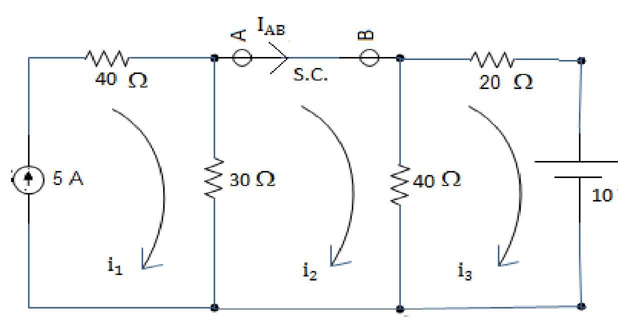


Fig. 4.25 (b)

Solution:

(1) Mark the terminal A-B across the load resistance R_L as shown in fig. 4.25 (b).

(2) Find I_N: For that apply the following three steps.

(i) Remove load resistance R_L and make the terminal A and B short circuited as shown in fig. 4.25 (b).

(ii) Find the current in all the meshes using mesh analysis.

In first mesh $\quad i_1 = 5 \quad$ ....(4.44)

In second mesh $\quad 30(i_1 - i_2) - 40(i_2 - i_3) = 0 \quad \Rightarrow \quad 30i_1 - 70i_2 + 40i_3 = 0 \quad$ ....(4.45)

In third mesh $\quad 40(i_2 - i_3) - 20i_3 - 10 = 0 \quad \Rightarrow \quad 40i_2 - 60i_3 = 10 \quad$ ....(4.46)

By solving the eq. 4.44, 4.45 and 4.46 $\quad i_1 = 5\text{ Amp.} \quad i_2 = 3.3\text{ Amp.} \quad i_3 = 2.04\text{ Amp.}$

Find the current between terminal A and B . $\quad I_{AB} = I_N = i_2 = 3.3\text{ Amp.}$

(3) Find R_N: For that apply the following three steps

(i) Remove load resistance R_L and make the terminal A and B open circuited as shown in fig. 4.26 (a).

(ii) Make all voltage source short circuited and all current source open circuited as shown in fig. 4.26 (a).

(iii) Now draw simplify circuit as shown in fig. 4.26 (b), By solving the circuit.

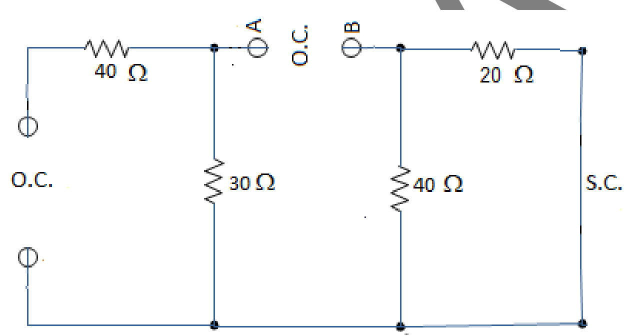


Fig. 4.26 (a)

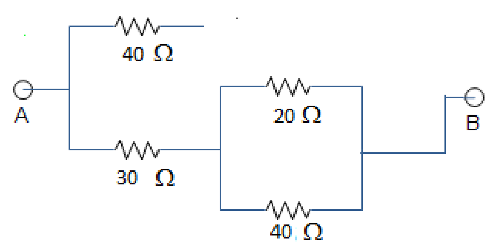


Fig. 4.26 (b)

$$R_{AB} = R_N = 30 + \frac{20 \times 40}{20 + 40} = 43.33\ \Omega$$

(4) Find V_{Th}: $\quad V_{Th} = I_N R_N = 3.3 \times 43.33 = 142.99\text{ V}$

(5) Find R_{Th}: $\quad R_{Th} = R_N = 43.33\ \Omega$

(6) The value of load resistance $\quad R_L = R_{Th} = 43.33\ \Omega \quad \textbf{Ans.}$

(7) The value of maximum power

$$P_{Lmax} = \frac{V_{Th}^2}{4R_{Th}} = \frac{142.99^2}{4 \times 43.33} = 117.97\ \text{W} \quad \textbf{Ans.}$$

Example 4.14 In the circuit shown in fig. 4.27 (a), Find the value of load resistance R_L if maximum power will transfer to the load resistance. Also find the value of the maximum power.

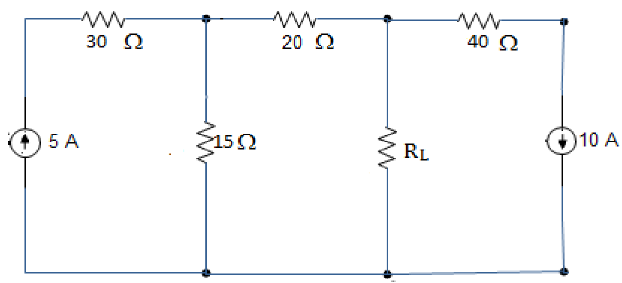


Fig. 4.27 (a)

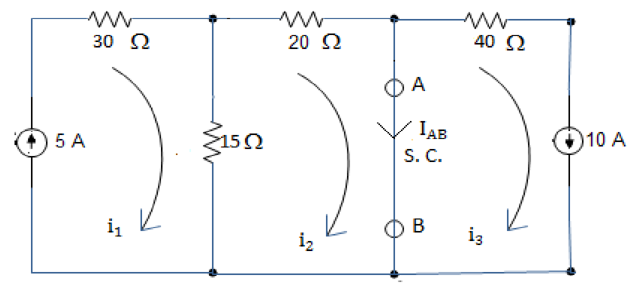


Fig. 4.27 (b)

Solution:

(1) Mark the terminal A-B across the load resistance R_L as shown in fig. 4.27 (b).

(2) Find I_N: For that apply the following three steps.

(i) Remove load resistance R_L and make the terminal A and B short circuited as shown in fig. 4.27 (b).

(ii) Find the current in all the meshes using mesh analysis.

In first mesh $\quad i_1 = 5$ ....(4.47)

In second mesh $\quad 15(i_1 - i_2) - 20i_2 = 0 \quad \Rightarrow \quad 15i_1 - 35i_2 = 0$ ....(4.48)

In third mesh $\quad i_3 = 10$ ....(4.49)

By solving the eq. 4.47, 4.48 and 4.49 $\quad i_1 = 5\ \text{Amp.} \quad i_2 = 2.14\ \text{Amp.} \quad i_3 = 10\ \text{Amp.}$

Find the current between terminal A and B . $\quad I_{AB} = I_N = i_2 - i_3 = 2.14 - 10 = -7.86\ \text{Amp.}$

(3) Find R_N: For that apply the following three steps

(i) Remove load resistance R_L and make the terminal A and B open circuited as shown in fig. 4.28 (a).

(ii) Make all voltage source short circuited and all current source open circuited as shown in fig. 4.28 (a).

(iii) Now draw simplify circuit as shown in fig. 4.28 (b), By solving the circuit.

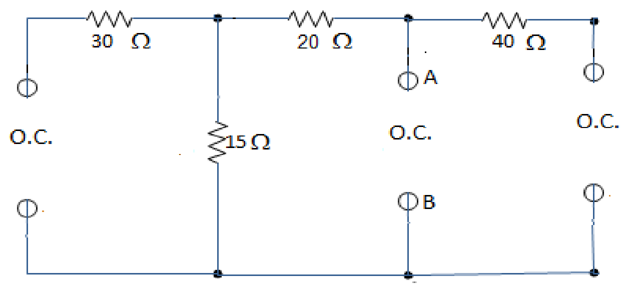


Fig. 4.28 (a)

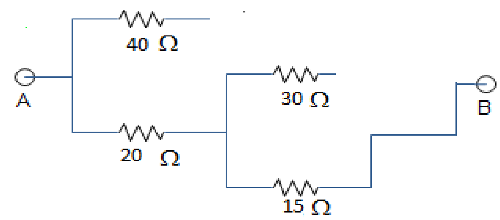


Fig. 4.28 (b)

$$R_{AB} = R_N = 20 + 15 = 35\,\Omega$$

(4) Find V_{Th}: $\quad V_{Th} = I_N R_N = -7.86 \times 35 = -275.1$

(5) Find R_{Th}: $\quad R_{Th} = R_N = 35\,\Omega$

(6) The value of load resistance $\quad R_L = R_{Th} = 35\,\Omega$ **Ans.**

(7) The value of maximum power

$$P_{Lmax} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(-275.1)^2}{4 \times 35} = 540.57\text{ W} \quad \textbf{Ans.}$$