UNIT 1

ELECTRICAL CIRCUIT ANALYSIS

1.1 CIRCUIT ELEMENT

It is the smallest part of the circuit. Electric circuit mainly consist of five element

(1) **Voltage Source:** Elements which provide voltage to the circuit is called the voltage source. The unit of the voltage is volt. The symbol of the voltage source is shown in fig. 1.1.

(2) **Current Source:** Elements which provide current to the circuit is called the current source. The unit of the current is Ampere (Amp.). The symbol of the current source is shown in fig. 1.1.

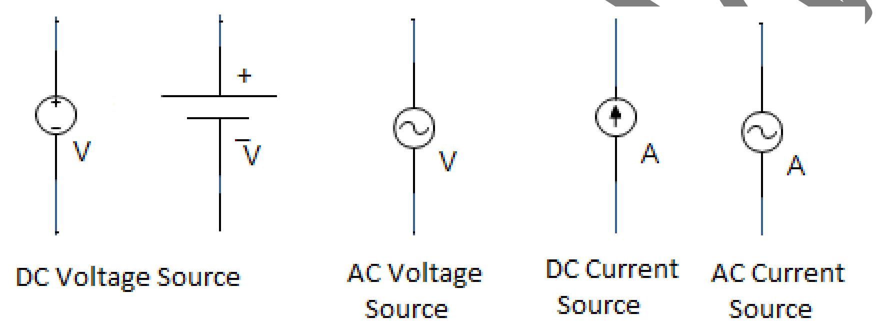


Fig. 1.1

(3) **Resistor:** Elements which oppose the flow of current is called the resistor. Resistance is the property of the resistor by which it opposes the flow of current. The unit of the resistance is ohm (Ω). The symbol of the resistor is shown in fig. 1.2 (a).

(4) **Inductor:** Element which store the energy in the form of magnetic field is called the inductor. Inductance is the property of the inductor by which it stores the energy in the form of magnetic field. The unit of the inductance is Henry (H). The symbol of the inductor is shown in fig. 1.2 (b).

(5) **Capacitor:** Element which store the energy in the form of electric field is called the capacitor. Capacitance is the property of the capacitor by which it stores the energy in the form of electric field. The unit of the capacitance is Faraday (F). The symbol of the capacitor is shown in fig. 1.2 (c).

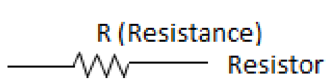


Fig. 1.2 (a)

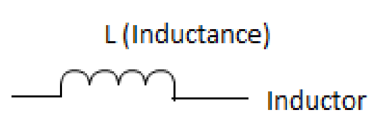


Fig. 1.2 (b)

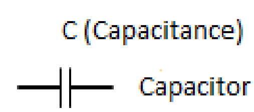


Fig. 1.2 (c)

Relation between the voltage, current and property of the element for resistor, inductor and capacitor:

Element	For instantaneous value	For RMS value	Power
Resistor	$v = Ri$	V = IR	P = I^2R = $\frac{V^2}{R}$
Inductor	$v = L\frac{di}{dt}$	V = IX_L	P = $\frac{1}{2}LI^2$
Capacitor	$v = \frac{1}{C}\int idt$	V = IX_C	P = $\frac{1}{2}CV^2$

Where

X_L= 2πfL is called inductive reactance. It is the opposition offered by the inductor to the flow of current. The unit of the X_L is ohm (Ω).

X_C= $\frac{1}{2\pi fC}$ is called capacitive reactance. It is the opposition offered by the capacitor to the flow of current. The unit of the X_L is ohm (Ω).

f is the supply frequency.

1.2 CLASSIFICATION OF THE ELEMENT

1.2.1 **Active and Passive Element:**

Active Element: Elements which provide energy to the circuit is called active element. Example: Voltage Source, Current Source

Passive Element: Elements which receive energy from the circuit is called the passive element. Example: Resistor, Inductor, Capacitor

1.2.2 **Unilateral and Bilateral Element:**

Unilateral Element: Elements whose behavior change with the change in the direction of the current is called unilateral element. Example: Diode

Bilateral Element: Elements whose behavior does not change with the change in the direction of the current is called bilateral element. Example: Resistor

1.2.3 **Linear and Nonlinear Element:**

Linear Element: Element which follow the ohm law and superposition theorem is called the linear element. Example: Resistor

Nonlinear Element: Element which does not follow the ohm law and superposition theorem is called the nonlinear element. Example: Diode

1.3 CIRCUIT CONCEPT

Network: Any combination of the circuit element is called the network.

Circuit: Any combination if the circuit element in which at least one active element should be there in the closed path is called the circuit. For circuit it is necessary current should be flowing.

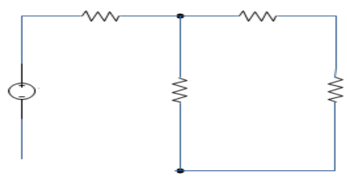


Fig 1.3 (a)

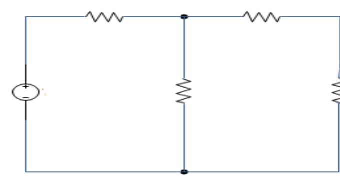


Fig. 1.3 (b)

Fig. 1.3 (a) is only network because there is no active element in the closed path. There is no flow of current

Fig. 1.3 (b) is network as well as circuit because there is an active element in the closed path. There is a flow of current

Note: All the circuits are network but the entire network are not circuit.

Loop: Any closed path in the circuit is called loop.

Mesh: Any closed path in the circuit which is not the combination of the closed path is called mesh.

Junction: A point where three or more than three element is connected is called junction.

Node: A point where two or more than two element is connected is called node.

Branch: The entire possible path between any two junctions in a circuit is called branch.

Example 1.1 Find the number of loop, mesh, junction, node and branch in the circuit shown in fig. 1.4.

Solution:

Number of Loop: 3 (ABEDA, BCFEB, ABCFEDA)

Number of Mesh: 2 (ABEDA, BCFEB)

Number of Junction: 2 (B, E)

Number of Node: 4 (A, B, C, E)

Number of Branch: 3 (BADE, BE, BCFE)

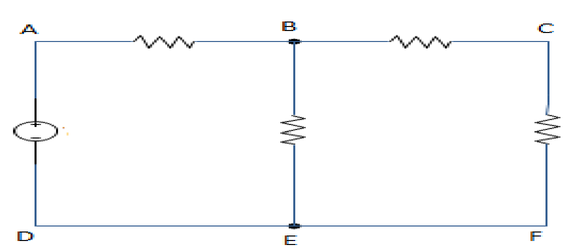


Fig. 1.4

Example 1.2 Find the number of loop, mesh, junction, node and branch in the circuit shown in fig. 1.5.

Solution:

Number of Loop: 7 (ABEDA, BCFEB, ABCHGA, ABCFEDA, AGHCBEDA, BAGHCFEB, AGHCFEDA)

Number of Mesh: 3 (ABEDA, BCFEB, ABCHGA)

Number of Junction: 4 (A, B, C, E)

Number of Node: 4 (A, B, C, E)

Number of Branch: 6 (AB, AGHC, ADE, BC, BE, CFE)

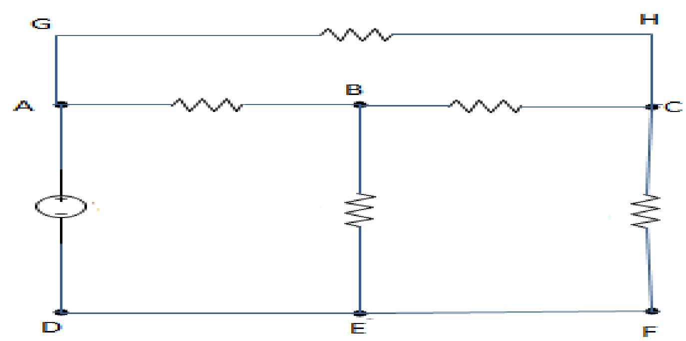


Fig. 1.5

Open Circuit: If there is no element between the two point in a circuit then there two points is called open circuited. In the case of open circuit the resistance between two points is infinite. Fig. 1.6 (a) is showing the condition of open circuit.

Short Circuit: If two points are connected by a wire without any resistance then these points is called short circuited. In the case of short circuit the resistance between two points is zero. Fig. 1.6 (b) is showing the condition of short circuit.

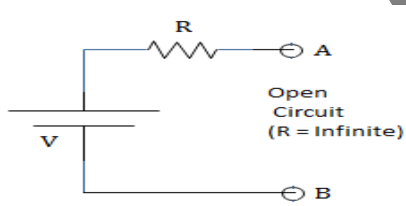


Fig. 1.6 (a)

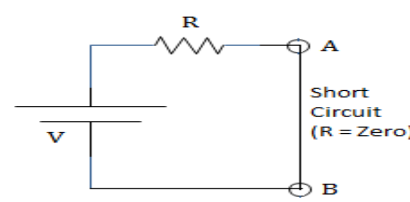


Fig. 1.6 (b)

1.4 CLASSIFICATION OF SOURCES

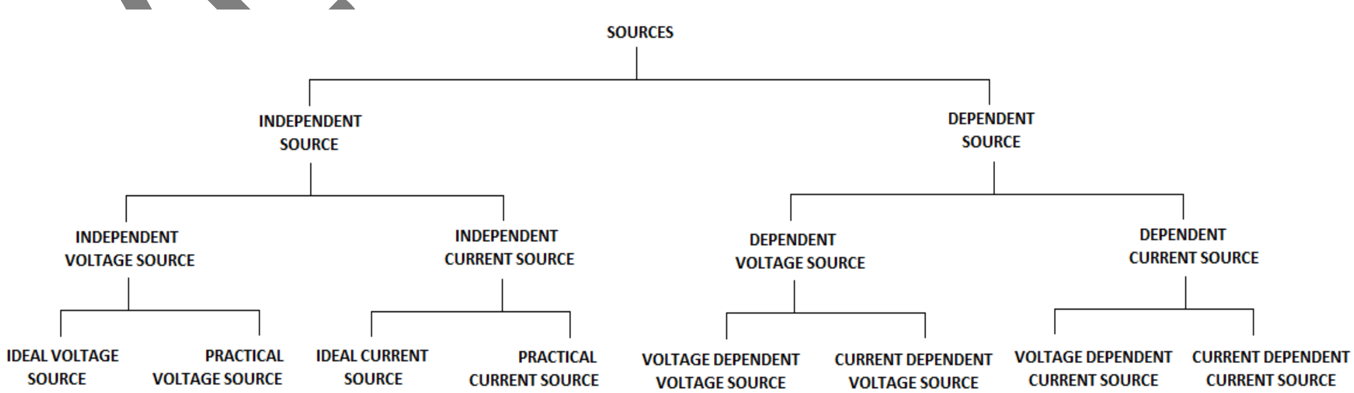


Fig. 1.7

1.4.1 **Independent source:** The source which does not depend on any other parameter of the circuit is called independent source. It is of two types

(i) Independent voltage source: In independent voltage source, source voltage is connected in series with internal resistance. The symbol of the independent voltage source is shown in fig. 1.8 (a).It is of two types

(a) Ideal voltage source: Independent voltage source will be ideal if $R_{in} = 0$ (Short Circuit). The symbol of the ideal voltage source is shown in fig. 1.8 (b)

(b) Practical voltage source: Independent voltage source will be practical if $R_{in} \neq 0$. The symbol of the practical voltage source shown in fig. 1.8 (c)

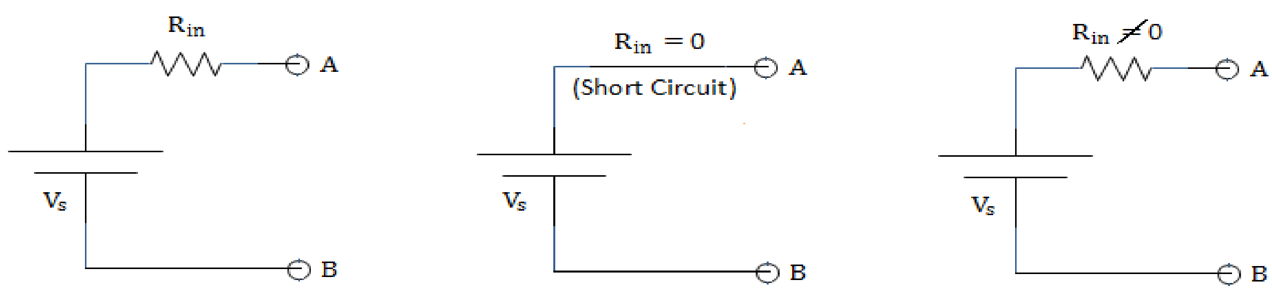


Fig. 1.8 (a) **Fig. 1.8 (b)** **Fig 1.8 (c)**

(ii) Independent current source: In independent current source, source current is connected in parallel with internal resistance. The symbol of the independent current source is shown in fig. 1.9 (a).It is of two types

(a) Ideal current source: Independent current source will be ideal if $R_{in} = \infty$ (open Circuit). The symbol of the ideal current source is shown in fig. 1.9 (b).

(b) Practical current source: Independent current source will be practical if $R_{in} \neq \infty$. The symbol of the practical current source is shown in fig. 1.9 (c).

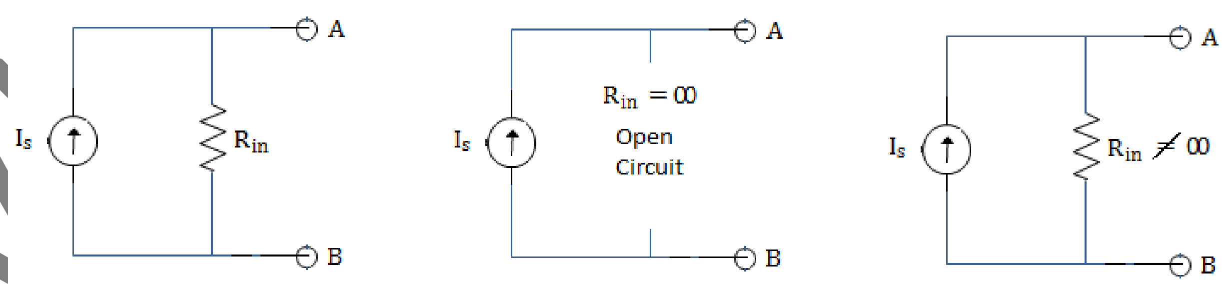


Fig. 1.9 (a) **Fig. 1.9 (b)** **Fig. 1.9 (c)**

The V-I characteristic of Ideal and practical voltage source is shown in fig. 1.10 (a).

The V-I characteristic of Ideal and practical current source is shown in fig. 1.10 (b).

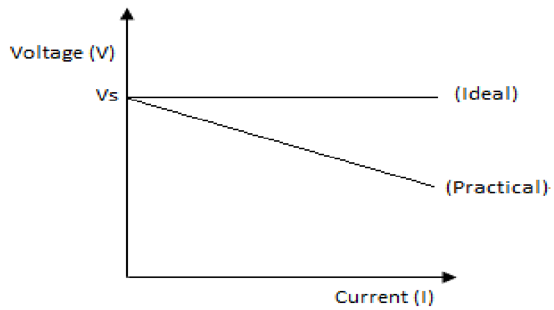


Fig. 1.10 (a)

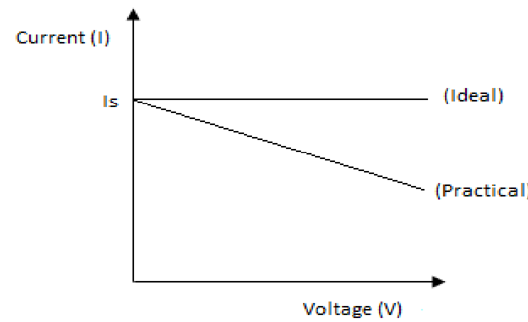


Fig. 1.10 (b)

1.4.2 **Dependent source:** The source which depends on any other parameter of the circuit is called dependent source. It is of two types

(i) Dependent voltage source: The voltage source which depends on any other parameter of the circuit is called dependent voltage source. It is of two types

(a) Voltage Dependent Voltage Source (VDVS): Voltage source which depends on any other voltage of the circuit is called voltage dependent voltage source. The symbol of VDVS is shown in fig. 1.11 (a)

(b) Current Dependent Voltage Source (CDVS): Voltage source which depends on any other current of the circuit is called current dependent voltage source. The symbol of CDVS is shown in fig. 1.11 (b)

(ii) Dependent current source: The current source which depends on any other parameter of the circuit is called dependent current source. It is of two types

(a) Voltage Dependent Current Source (VDCS): Current source which depends on any other voltage of the circuit is called voltage dependent current source. The symbol of VDCS is shown in fig. 1.11 (c)

(b) Current Dependent Current Source (CDCS): Current source which depends on any other current of the circuit is called current dependent current source. The symbol of CDCS is shown in fig. 1.11 (d)

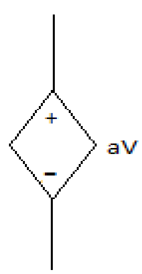


Fig. 1.11 (a)

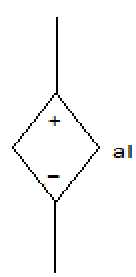


Fig. 1.11 (b)

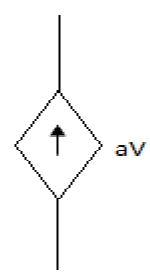


Fig. 1.11 (c)

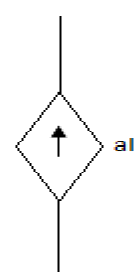


Fig 1.11 (d)

1.5 SOURCE TRANSFORMATION

The conversion of voltage source into current source or vice-versa is called the source transformation.

1.5.1 **Voltage source to current source:** A Practical voltage source shown in fig. 1.12 (a) is transform into practical current source shown in fig. 1.12 (b).

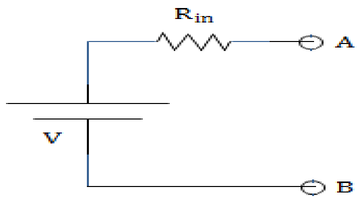


Fig. 1.12 (a)

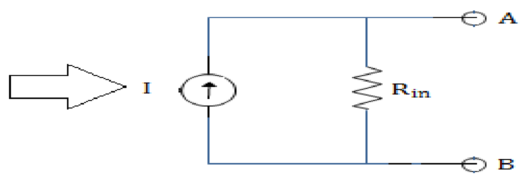


Fig. 1.12 (b)

In that case you know the value of V and $\mathrm{R_{in}}$ and you have to find the value of I and $\mathrm{R_{in}}$.

Find the I: using ohm law $\quad I = \frac{V}{R_{in}}$

Find the $\mathrm{R_{in}}$: $\quad\mathrm{R_{in}}$ of current source = $\mathrm{R_{in}}$ of voltage source

1.5.2 **Current source to voltage source:** A Practical current source shown in fig. 1.13 (a) is transformed into practical voltage source shown in fig. 1.13 (b).

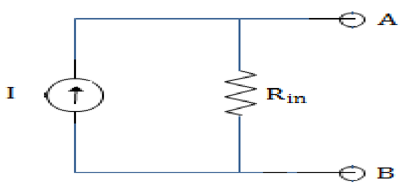


Fig. 1.13 (a)

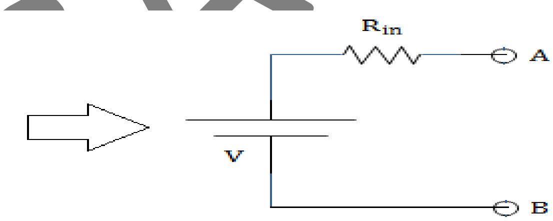


Fig. 1.13 (b)

In that case you know the value of I and $\mathrm{R_{in}}$ and you have to find the value of V and$\mathrm{R_{in}}$.

Find the V: using ohm law $\quad V = IR_{in}$

Find the $\mathrm{R_{in}}$: $\quad\mathrm{R_{in}}$ of voltage source = $\mathrm{R_{in}}$ of current source

Note: Ideal sources and dependent sources are not transformable.

1.6 CONCEPT FOR SOLVING THE CIRCUIT

Some concepts for solving the circuit are as follow.

1.6.1 **Voltage across resistance:**

If a current I flowing in a resistance R as shown in fig 1.14, then voltage across the resistance is given by

$$V = IR$$

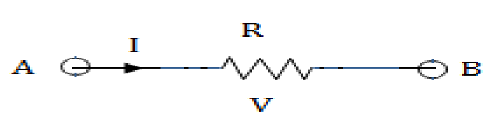


Fig. 1.14

1.6.2 **Current in resistance:**

If voltage of terminal A is V_1 and voltage of terminal B is V_2 across a resistance R as shown in fig.1.15, then the current in resistance R is given by

$$I = \frac{V_1 - V_2}{R}$$

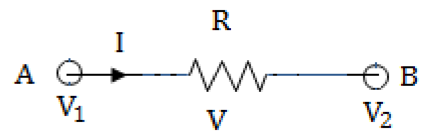


Fig. 1.15

1.6.3 **Resistor in series:** Two resistance R_1 and R_2 is connected in series between terminal A and B as shown in fig. 1.16

Then total resistance between terminal A and B is $R = R_1 + R_2$

In series resistance voltage is divided and current is same.

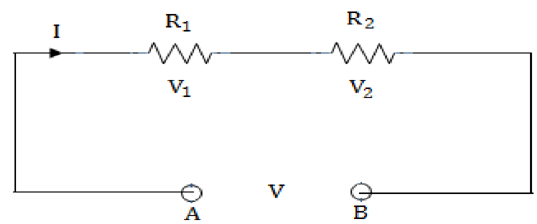


Fig. 1.16

The total current is given by

$$I = \frac{V}{R} = \frac{V}{R_1 + R_2}$$

The voltage across resistance R_1 is

$$V_1 = IR_1 = \frac{VR_1}{R_1 + R_2}$$

The voltage across resistance R_2 is

$$V_2 = IR_2 = \frac{VR_2}{R_1 + R_2}$$

1.6.4 **Resistor in parallel:** Two resistance R_1 and R_2 is connected in parallel between terminal A anb B as shown in fig. 1.17

The total resistance between terminal A and B is

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

In parallel resistance voltage is same and current is divided.

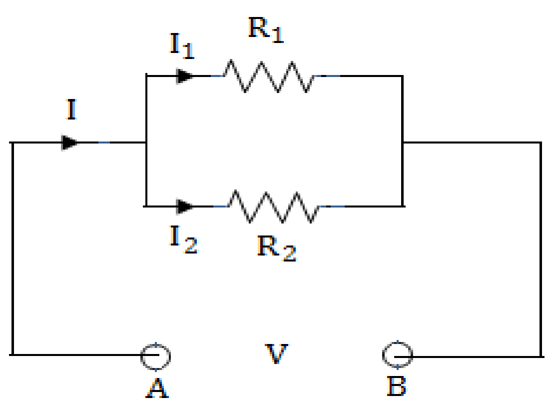


Fig. 1.17

The total current is given by

$$I = \frac{V}{R} = \frac{V}{\frac{R_1 R_2}{R_1 + R_2}} \Rightarrow V = \frac{IR_1 R_2}{R_1 + R_2}$$

The current in resistance R_1 is

$$I_1 = \frac{V}{R_1} = \frac{IR_2}{R_1 + R_2}$$

The current in resistance R_2 is

$$I_2 = \frac{V}{R_2} = \frac{IR_1}{R_1 + R_2}$$

1.7 STAR AND DELTA CONNECTION

If we connect three resistances with each other, other than series and parallel two more connection is possible that is called star and delta connection.

1.7.1 **Star connection:** if the second terminal of three resistances is join together that connection is called star connection as shown in fig. 1.18 (a). R_A, R_B and R_C is the resistance of the star connection connected to terminal A, B and C respectively

1.7.2 **Delta connection:** If the second terminal of the one resistance is connected to first terminal of the another resistance and forming a closed path that connection is called delta connection as shown in fig. 1.18 (b). R_{AB}, R_{BC} and R_{CA} is the resistance of the delta connection connected between terminal AB, BC and CA respectively

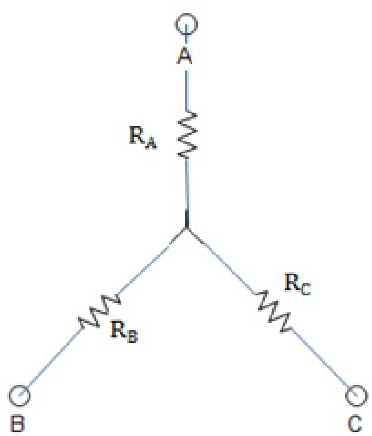


Fig. 1.18 (a)

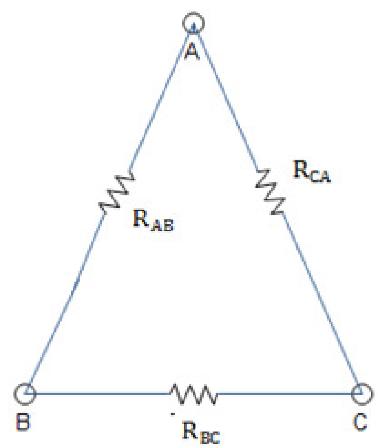


Fig. 1.18 (b)

1.8 STAR DELTA TRANSFORMATION

For conversion of star connection into delta connection or delta connection into star connection is called star delta transformation.

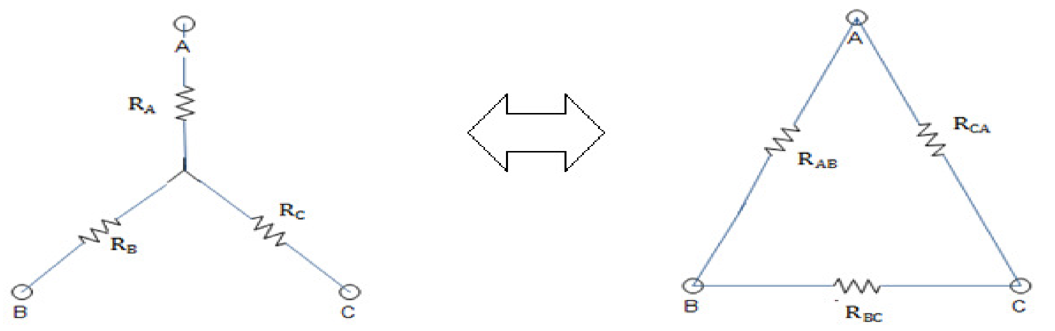


Fig. 1.19

In star delta transformation we know the value of star resistance (R_A, R_B and R_C) and with the help of these resistances we have to find the value of delta resistance(R_{AB}, R_{BC} and R_{CA}).

In delta star transformation we know the value of delta resistance (R_{AB}, R_{BC} and R_{CA}).and with the help of these resistances we have to find the value of star resistance (R_A, R_B and R_C)

Consider the terminal A and B in fig. 1.19

For star connection the resistance R_A and R_B will come in series, So total resistance between terminal A and B in fig. 1.19 for star connection = $R_A + R_B$

For delta connection the resistance the resistance R_{BC} and R_{CA} will come in series and R_{AB} will be in parallel with($R_{BC} + R_{CA}$), So total resistance between terminal A and B in fig. 1.19 for delta connection =

$$\frac{R_{AB}(R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}}$$

For transformation total resistance between terminal A and B for both the connection should be equal

$$R_A + R_B = \frac{R_{AB}(R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}}$$

….(1.1)

Similarly between terminal B and C

$$R_B + R_C = \frac{R_{BC}(R_{AB} + R_{CA})}{R_{BC} + R_{AB} + R_{CA}}$$

….(1.2)

Similary between terminal C and A

$$R_C + R_A = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{CA} + R_{AB} + R_{BC}}$$

....(1.3)

By adding equation (1.1), (1.2) and (1.3)

$$R_A + R_B + R_B + R_C + R_C + R_A = \frac{R_{AB}(R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} + \frac{R_{BC}(R_{AB} + R_{CA})}{R_{BC} + R_{AB} + R_{CA}} + \frac{R_{CA}(R_{AB} + R_{BC})}{R_{CA} + R_{AB} + R_{BC}}$$

$$\Rightarrow \qquad 2R_A + 2R_B + 2R_C = \frac{R_{AB}R_{BC} + R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} + \frac{R_{BC}R_{AB} + R_{BC}R_{CA}}{R_{BC} + R_{AB} + R_{CA}} + \frac{R_{CA}R_{AB} + R_{CA}R_{BC}}{R_{CA} + R_{AB} + R_{BC}}$$

$$\Rightarrow \qquad 2(R_A + R_B + R_C) = \frac{2R_{AB}R_{BC} + 2R_{AB}R_{CA} + 2R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$\Rightarrow \qquad R_A + R_B + R_C = \frac{R_{AB}R_{BC} + R_{AB}R_{CA} + R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

....(1.4)

By subtracting eq. (1.1) from eq. (1.4)

$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

....(1.5)

By subtracting eq. (1.2) from eq. (1.4)

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

....(1.6)

By subtracting eq. (1.3) from eq. (1.4)

$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

....(1.7)

With the help of equation (1.5), (1.6) and (1.7) if we know the value of delta resistances (R_{AB}, R_{BC} and R_{CA}) we can find the value of star resistances (R_A, R_B and R_C). So eq. (1.5), (1.6) and (1.7) represent the delta to star transformation.

Divide the eq. (1.6) from eq. (1.5)

$$\frac{R_C}{R_A} = \frac{R_{BC}}{R_{AB}} \Rightarrow R_{BC} = \frac{R_C R_{AB}}{R_A}$$

Divide the eq. (1.7) from eq. (1.5)

$$\frac{R_C}{R_B} = \frac{R_{CA}}{R_{AB}} \Rightarrow R_{CA} = \frac{R_C R_{AB}}{R_B}$$

Substitute the value of R_{BC} and R_{CA} in eq. (1.5)

$$R_C = \frac{\frac{R_C R_{AB}}{R_A}\frac{R_C R_{AB}}{R_B}}{R_{AB} + \frac{R_C R_{AB}}{R_A} + \frac{R_C R_{AB}}{R_B}}$$

$$\Rightarrow \qquad R_C = \frac{\frac{R_C^2 R_{AB}^2}{R_A R_B}}{R_{AB}\left(1 + \frac{R_C}{R_A} + \frac{R_C}{R_B}\right)}$$

$$\Rightarrow \qquad R_C = \frac{\frac{R_C^2 R_{AB}^2}{R_A R_B}}{R_{AB}\left(\frac{R_A R_B + R_C R_B + R_C R_A}{R_A R_B}\right)}$$

$$\Rightarrow \qquad R_C = \frac{R_C^2 R_{AB}}{R_A R_B + R_C R_B + R_C R_A}$$

$$\Rightarrow \qquad R_{AB} = \frac{R_A R_B + R_C R_B + R_C R_A}{R_C}$$

....(1.8)

Similarly

$$R_{BC} = \frac{R_A R_B + R_C R_B + R_C R_A}{R_A}$$

....(1.9)

Similarly

$$R_{CA} = \frac{R_A R_B + R_C R_B + R_C R_A}{R_B}$$

....(1.10)

With the help of equation (1.8),(1.9) and (1.10) if we know the value of star resistances (R_A, R_B and R_C) we can find the value of delta resistances (R_{AB}, R_{BC} and R_{CA}). So eq. (1.8), (1.9) and (1.10) represent the star to delta transformation.

Example 1.3 Find the total resistance between the terminal A and B shown in fig. 1.20.

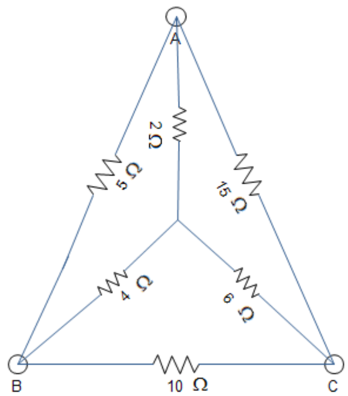


Fig. 1.20

Solution: Consider the resistance 2 Ω, 4 Ω and 6 Ω which is connected in star and converted into delta by using star delta transformation eq. no. (1.8), (1.9) and (1.10) as shown in fig. 1.21.

$$R_{AB} = \frac{R_A R_B + R_C R_B + R_C R_A}{R_C}$$

$$= \frac{2\times 4\ +\ 4\times 6\ +\ 6\times 2}{6} = \frac{44}{6} = 7.33$$

$$R_{BC} = \frac{R_A R_B + R_C R_B + R_C R_A}{R_A}$$

$$= \frac{2\times 4\ +\ 4\times 6\ +\ 6\times 2}{2} = \frac{44}{2} = 22\ \Omega$$

$$R_{CA} = \frac{R_A R_B + R_C R_B + R_C R_A}{R_B}$$

$$= \frac{2\times 4\ +\ 4\times 6\ +\ 6\times 2}{4} = \frac{44}{4} = 11\ \Omega$$

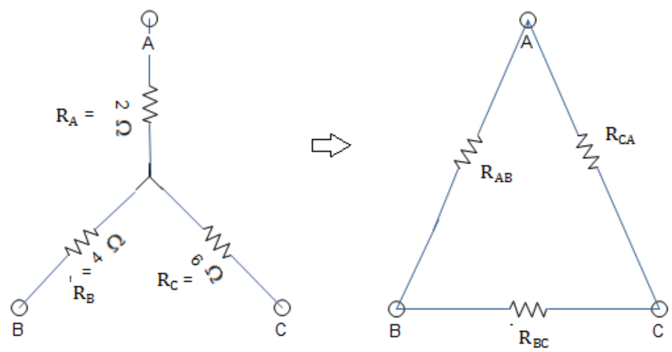


Fig. 1.21

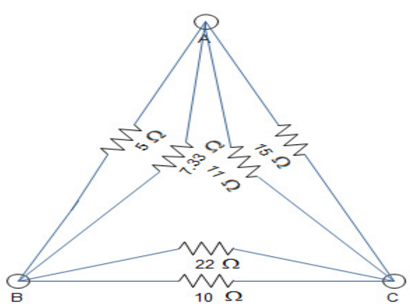


Fig. 1.22 (a)

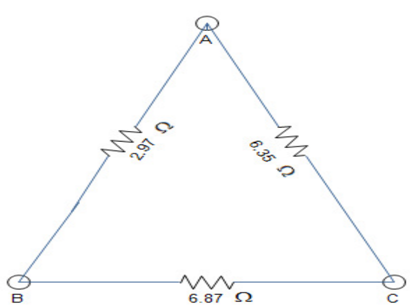


Fig. 1.22 (b)

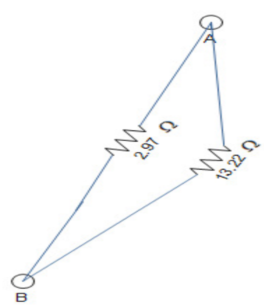


Fig. 1.22 (c)

Now replace the star in to delta in the original circuit as shown in fig. 1.22 (a).

Now 5 Ω is parallel with 7.33 Ω, 10 Ω is parallel with 22 Ω and 15 Ω is parallel with 11 Ω, solve it and convert it into a single delta as shown in fig. 1.22 (b)

Now between the terminal A and B 6.35 Ω and 6.87 Ω are in series and this series combination is in parallel with 2.97 Ω as shown in fig. 1.22 (c)

Now the total resistance between terminal A and B is $R_{AB} = 2.42\ \Omega$ **Ans**.

Example 1.4 Find the total resistance between the terminal A and B shown in fig. 1.23.

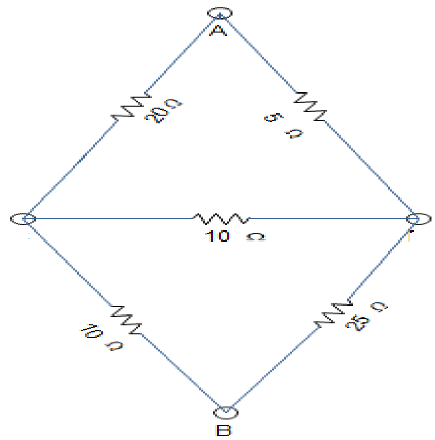


Fig. 1.23

Solution: Consider the resistance 20 Ω, 10 Ω and 5 Ω which are connected in delta and convert it into star by using delta star transformation eq. no. (1.5), (1.6) and (1.7) as shown in fig. 1.24.

$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} = \frac{10 \times 5}{20 + 10 + 5}$$
$$= \frac{50}{35} = 1.43\ \Omega$$
$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} = \frac{20 \times 5}{20 + 10 + 5}$$
$$= \frac{100}{35} = 2.86\ \Omega$$
$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}} = \frac{20 \times 10}{20 + 10 + 5}$$
$$= \frac{200}{35} = 5.71\ \Omega$$

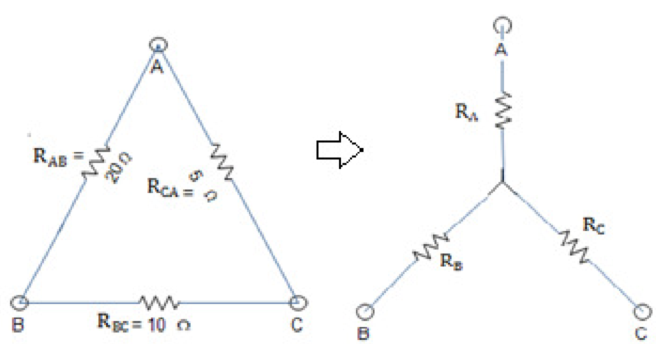


Fig. 1.24

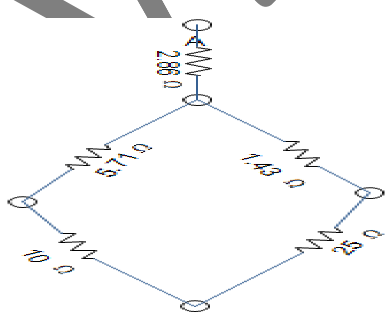


Fig.1.25 (a)

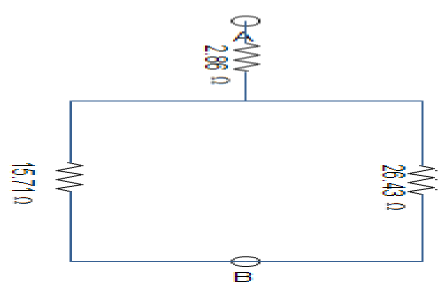


Fig. 1.25 (b)

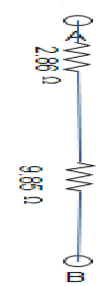


Fig. 1.25 (c)

Now replace the delta by star in the original circuit as shown in fig. 1.25 (a).

Now the resistance 10 Ω and 5.71 Ω or 25 Ω and 1.43 Ω are in series as shown in fig. 1.25 (a).

Now 15.71 Ω and 26.43 Ω are in parallel as shown in fig 1.25 (b).

Now 2.86 Ω and 9.85 Ω are in series as shown in fig. 1.25 (c).

So the total resistance between terminal A and B is $\mathrm{R_{AB}} = 12.71\ \Omega$ **Ans.**

1.9 KIRCHHOFF'S LAWS

G. R. Kirchhoff given two basic law for simplifying the circuit. First law is known as Kirchhoff current law and second law is known as Kirchhoff voltage law.

1.9.1 **Kirchhoff Current Law (KCL):** According to this law "At any point the algebraic sum of all the current is always zero". This law is based on the principle of conservation of charge.

Steps for apply KCL

(1) Consider the outgoing current from the point is positive and incoming current to the point is negative.

(2)Now take the sum of all the current at the point with proper sign.

The equation of KCL is for fig. 1.26 is

$$\mathrm{i_1 - i_2 - i_3 - i_4 - i_5 + i_6 = 0}$$

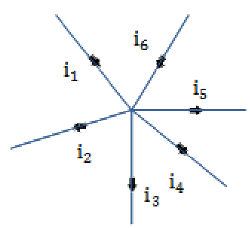


Fig. 1.26

1.9.2 **Kirchhoff Voltage Law (KVL):** According to this law "In any closed path the algebraic sum of all the voltage is always zero". This law is based on the principle of conservation of energy.

Steps for apply KVL

(1) Assume the current in clockwise direction in closed path.

(2) Mark the sign for each element: For voltage source bigger plate is always positive and smaller plate is always negative. For resistance the side is positive from where the assume current is entering and the side is negative from where the assume current is leaving.

(3) Now move in the closed path in clockwise direction take the sum of voltage across the element by considering the sign of the side from where you leaving the element.

The equation of KCL is for fig. 1.27 is

$$V - iR_1 - iR_2 - iR_3 = 0$$

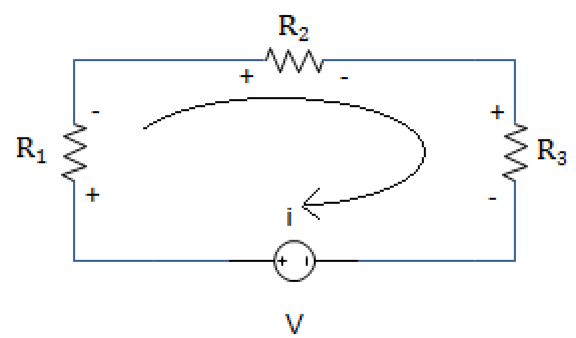


Fig. 1.27

1.10 MESH ANALYSIS (LOOP ANALYSIS)

It is a method to find the current in each mesh.

Steps for mesh analysis

(1) Find the number of mesh in the circuit.

(2) Assume a current in each mesh in clockwise direction.

(3) Mark the suitable sign for each element.

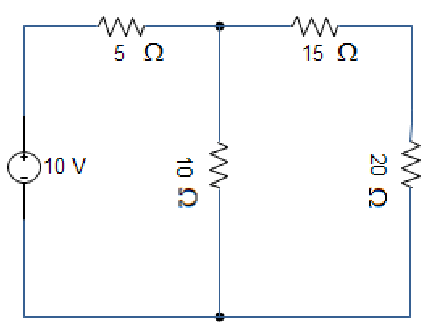
(4) Apply the KVL in each mesh.

(5) By solving the equation find the current in each mesh.

Mesh Analysis consist of three types of problem

1.10.1 **The circuit contains only voltage source and resistance:** In this type of problem apply the step of mesh analysis.

Example 1.5 Find the current in 10 Ω resistance shown in fig. 1.28 (a) using mesh analysis.



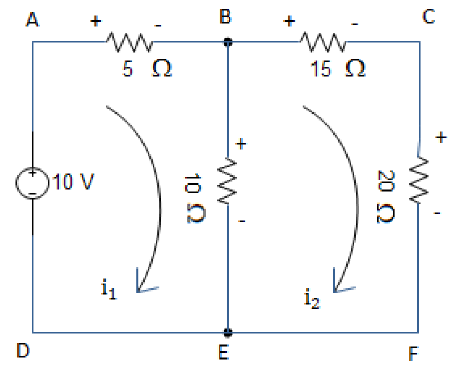


Fig. 1.28 (a) **Fig 1.28 (b)**

Solution: The circuit shown in fig. 1.28 (a) consists of two meshes (ABEDA, BCFEB), assume the current in both the mesh in clockwise direction as shown in fig. 1.28 (b).

The current in 5 Ω resistance is only i_1.The sign for 5 Ω resistance is mark by i_1, the side from where i_1 is entering in 5 Ω is positive and the side from where i_1, is leaving from 5 Ω is negative as shown in fig. 1.28 (b). The voltage across the 5 Ω resistance is $5i_1$.

The current in 15 Ω and 20 Ω resistance is only i_2. The sign for 15 Ω and 20 Ω resistance is mark by i_2, the side from where i_2 is entering in 15 Ω and 20 Ω is positive and the side from where i_2, is leaving from 15 Ω and 20 Ω is negative as shown in fig. 1.28 (b). The voltage across the 15 Ω and 20 Ω resistances is $15i_2$ and $20i_2$ respectively.

The current in 10 Ω resistance is both i_1 and i_2. The current $i_1$is flowing downward and current i_2 is flowing upward. Assume $i_1 > i_2$ so current in 10 Ω resistance is $(i_1 - i_2)$ in downward direction. The sign for 10 Ω resistance is mark by $(i_1 - i_2)$, the side from where $(i_1 - i_2)$ is entering in 10 Ω is positive and the side from where $(i_1 - i_2)$, is leaving from 10 Ω is negative as shown in fig. 1.28 (b). The voltage across the 10 Ω resistance is $10(i_1 - i_2)$.

Apply the KVL in mesh ABEDA

$$10 - 5i_1 - 10(i_1 - i_2) = 0$$

$$-15i_1 + 10i_2 = -10$$

….(1.11)

Apply the KVL in mesh BCFEB

$$10(i_1 - i_2) - 15i_2 - 20i_2 = 0$$

$$10i_1 - 45i_2 = 0$$

….(1.12)

By solving eq. (1.11) and (1.12), we have $i_1 = 0.7826 \text{ Amp.} \quad i_2 = 0.1739 \text{ Amp.}$

The current in 10 Ω resistance is = $i_1 - i_2 = 0.7826 - 0.1739 = 0.609 \text{ Amp. } \mathbf{Ans}.$

1.10.2 **The circuit contain current source which is associated with only one mesh:** In this case do not try to apply the KVL in the mesh containing current source. The current in that mesh is equal to the value of current source. If the direction of assume current is same as the direction of current source then the current is positive. If the direction of assume current is opposite as the direction of current source then the current is negative.

Example 1.6 Find the current in 20 Ω resistance shown in fig. 1.29 (a) using mesh analysis.

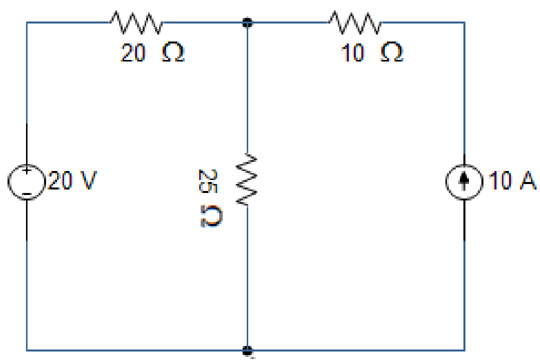


Fig. 1.29 (a)

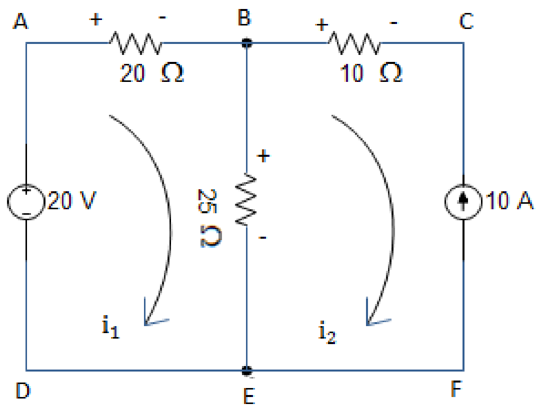


Fig. 1.29 (b)

Solution: The circuit shown in fig. 1.29 (a) consists of two meshes (ABEDA, BCFEB), assume the current in both the mesh in clockwise direction as shown in fig. 1.29 (b).

The second mesh consist a current source, so KVL is not applicable in second mesh. But because current source is only associated with second mesh, so the value of current in second mesh is equal to the value of current source. But the direction of i_2 and current source is opposite, so

$$i_2 = -10 \qquad ....(1.13)$$

Apply the KVL in mesh ABEDA

$$20 - 20i_1 - 25(i_1 - i_2) = 0$$

$$-45i_1 + 25i_2 = -20 \qquad ....(1.14)$$

By solving eq. (1.13) and (1.14), we have $i_1 = -5.11 \text{ Amp.} \quad i_2 = -10 \text{ Amp.}$

The current in 20 Ω resistance is = $i_1 = -5.11 \text{ Amp.}$ **Ans.**

1.10.3 **The circuit contain current source which is common between two meshes:** In this case do not try to apply KVL in any mesh in which current source is common. The first eq. is obtain by apply the KVL in the loop by combining both the mesh. The second eq. is obtained from the branch which contain current source. The value of current in that branch is equal to the value of current source.

Example 1.7 Find the current in 25 Ω resistance shown in fig. 1.30 (a) using mesh analysis.

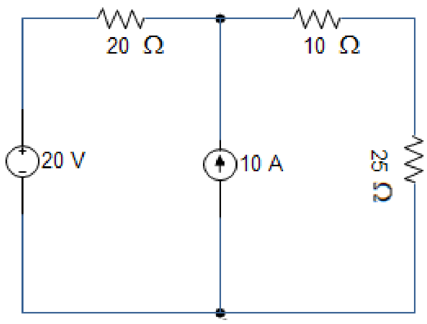


Fig. 1.30 (a)

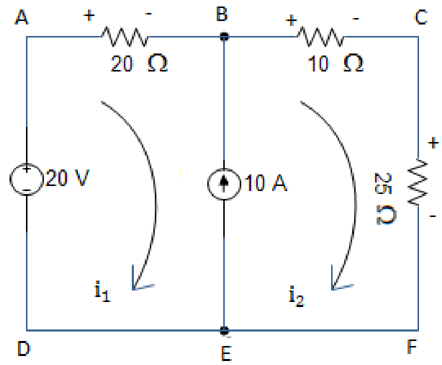


Fig. 1.30 (b)

Solution: The circuit shown in fig. 1.30 (a) consists of two meshes (ABEDA, BCFEB), assume the current in both the mesh in clockwise direction as shown in fig. 1.30 (b).

The 10 A current source is common between first and second mesh, so do not KVL apply in both the meshes.

Apply the KVL in loop ABCFEDA

$$20 - 20i_1 - 10i_2 - 25i_2 = 0$$

$$-20i_1 - 35i_2 = -20$$

....(1.15)

The current in the branch BE is equal to the value of current source. The direction is opposite, so

$$i_1 - i_2 = -10$$

....(1.16)

By solving eq. (1.15) and (1.16), we have $i_1 = -6 \text{ Amp.} \quad i_2 = 4 \text{ Amp.}$

The current in 25 Ω resistance is = $i_2 = 4 \text{ Amp.}$ **Ans.**

Example 1.8 Find the current in 25 Ω and 30 Ω resistance shown in fig. 1.31 (a) using mesh analysis.

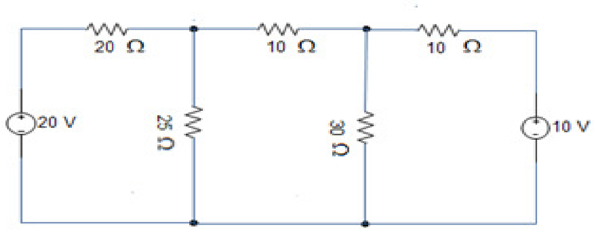


Fig. 1.31 (a)

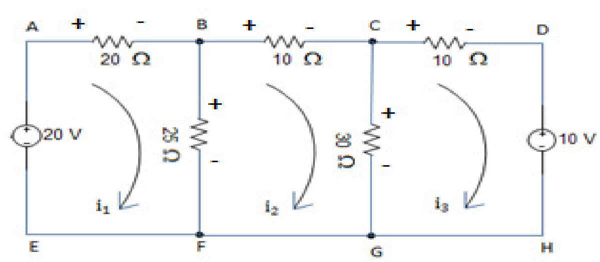


Fig. 1.31 (b)

Solution: The circuit shown in fig. 1.31 (a) consists of three meshes (ABFEA, BCGFB,CDHGC). The circuits redraw with mesh current and proper sign is shown in fig. 1.31 (b). Applying KVL to all the three meshes we have

In mesh ABFEA

$$20 - 20i_1 - 25(i_1 - i_2) = 0$$

$$-45i_1 + 25i_2 = -20$$

………… (1.17)

In mesh BCGFB

$$25(i_1 - i_2) - 10i_2 - 30(i_2 - i_3) = 0$$

$$25i_1 - 65i_2 + 30i_3 = 0$$

………… (1.18)

In mesh CDHGC

$$30(i_2 - i_3) - 10i_3 - 10 = 0$$

$$30i_2 - 40i_3 = 10$$

………… (1.19)

By solving eq. (1.17) ,(1.18) and (1.19), we have $i_1 = 0.514$ Amp. $i_2 = 0.126$ Amp. $i_3 = -0.155$ Amp.

The current in 25 Ω resistance is = $i_1 - i_2 = 0.514 - 0.126 = 0.388$ Amp. **Ans.**

The current in 30 Ω resistance is = $i_2 - i_3 = 0.126 - (-0.155) = 0.126 + 0.155 = 0.281$ Amp. **Ans.**

Example 1.9 Find the current in 10 Ω resistance shown in fig. 1.32 (a) using mesh analysis.

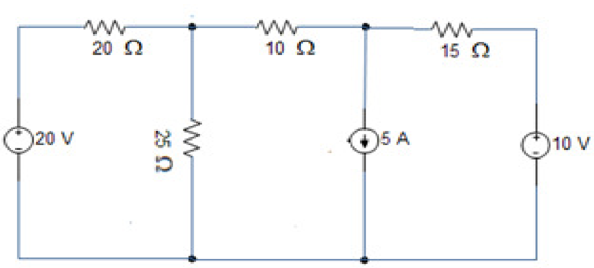


Fig. 1.32 (a)

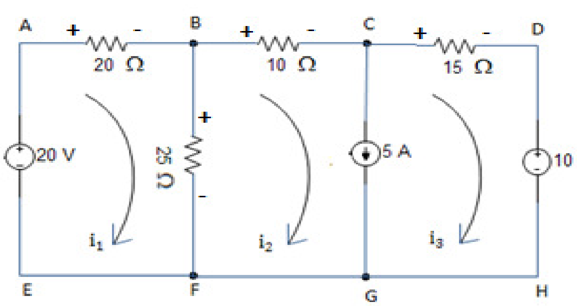


Fig. 1.32 (b)

Solution: The circuit shown in fig. 1.32 (a) consists of three meshes (ABFEA, BCGFB, and CDHGC). The circuits redraw with mesh current and proper sign is shown in fig. 1.32 (b). This problem consist current source between 2nd and 3rd mesh. By applying mesh analysis,

In mesh ABFEA

$$20 - 20i_1 - 25(i_1 - i_2) = 0$$

$$-45i_1 + 25i_2 = -20$$

…. (1.20)

For loop BCDHGFB

$$25(i_1 - i_2) - 10i_2 - 15i_3 - 10 = 0$$

$$25i_1 - 35i_2 - 15i_3 = 10$$

…. (1.21)

For branch CG

$$i_2 - i_3 = 5$$

…. (1.22)

By solving eq. (1.20) ,(1.21) and (1.22), we have $i_1 = 1.615$ Amp. $i_2 = 2.107$ Amp. $i_3 = -2.892$ Amp.

The current in 10 Ω resistance is = $i_2 = 2.107$ Amp. **Ans**.

Example 1.10 Find the current in 15 Ω resistance shown in fig. 1.33 (a) using mesh analysis.

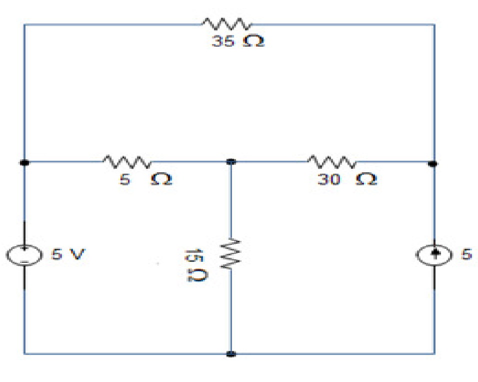


Fig. 1.33 (a)

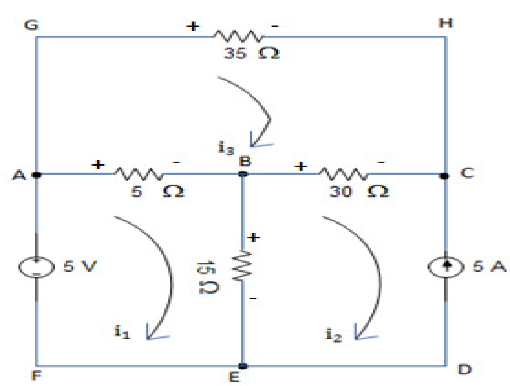


Fig. 1.33 (b)

Solution: The circuit shown in fig. 1.33 (a) consists of three meshes (ABFEA, BCDEB, and AGHCBA). The circuits redraw with mesh current and proper sign is shown in fig. 1.32 (b). This problem consist current source in mesh 2. By applying mesh analysis, we have

In mesh ABEFA

$$5 - 5(i_1 - i_3) - 15(i_1 - i_2) = 0$$

$$-20i_1 + 15i_2 + 5i_3 = -5$$

.... (1.23)

In mesh BCDEB

$$i_2 = -5$$

.... (1.24)

In mesh AGHCBA

$$5(i_1 - i_3) - 35i_3 + 30(i_2 - i_3) = 0$$

$$5i_1 + 30i_2 - 70i_3 = 0$$

.... (1.25)

By solving eq. (1.23) ,(1.24) and (1.25), we have $i_1 = -4.109$ Amp. $i_2 = -5$ Amp. $i_3 = -2.436$ Amp.

The current in 15 Ω resistance is = $i_1 - i_2 = -4.109 - (-5) = -4.109 + 5 = 0.891$ Amp. **Ans.**

Example 1.11 Find the current in 5 Ω resistance shown in fig. 1.34 (a) using mesh analysis.

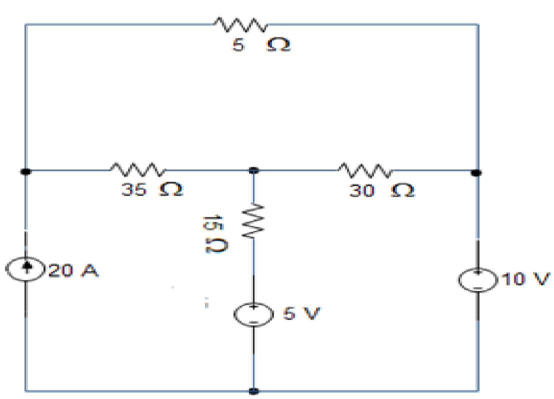


Fig. 1.34 (a)

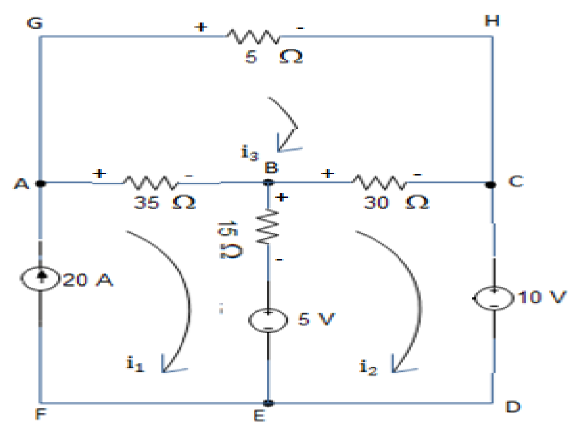


Fig. 1.34 (b)

Solution: This problem consist current source in mesh 1. By applying mesh analysis in fig. 1.34 (b),

$$i_1 = 20$$

.... (1.26)

$$5 + 15(i_1 - i_2) - 30(i_2 - i_3) = 0$$

$$15i_1 - 45i_2 + 30i_3 = -5$$

.... (1.27)

$$35(i_1 - i_3) - 5i_3 + 30(i_2 - i_3) = 0$$

$$35i_1 + 30i_2 - 70i_3 = 0$$

.... (1.28)

By solving eq. (1.26) ,(1.27) and (1.28), we have $i_1 = 20$ Amp. $i_2 = 18.82$ Amp. $i_3 = 18.07$ Amp.

The current in 15 Ω resistance is = $i_3 = 18.07$ Amp. **Ans.**

Example 1.12 Find the current in 15 Ω resistance shown in fig. 1.35 (a) using mesh analysis.

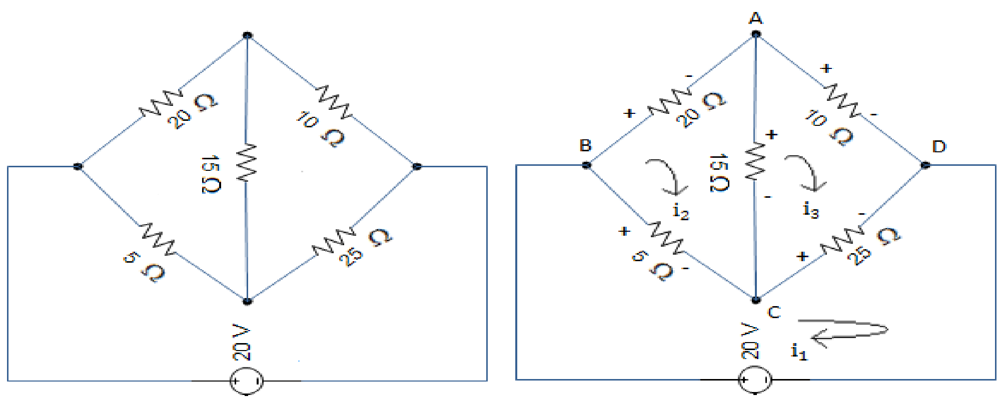


Fig. 1.35 (a) **Fig. 1.35 (b)**

Solution: By applying mesh analysis in fig. 1.35 (b),

$$20 - 5(i_1 - i_2) - 25(i_1 - i_3) = 0$$

$$-30i_1 + 5i_2 + 25i_3 = -20$$

....(1.29)

$$-20i_2 - 15(i_2 - i_3) + 5(i_1 - i_2) = 0$$

$$5i_1 - 40i_2 + 15i_3 = 0$$

....(1.30)

$$15(i_2 - i_3) - 10i_3 + 25(i_1 - i_3) = 0$$

$$25i_1 + 15i_2 - 50i_3 = 0$$

....(1.31)

By solving eq. (1.29) ,(1.30) and (1.31), we have $i_1 = 1.527\text{ Amp.}$ $i_2 = 0.538\text{ Amp.}$ $i_3 = 0.925\text{ Amp.}$

The current in 15 Ω resistance is = $i_2 - i_3 = 0.538 - 0.925 = -0.387\text{ Amp.}\,\mathbf{Ans}.$

1.11 NODAL ANALYSIS (JUNCTION ANALYSIS)

It is a method to find the voltage at each node.

Steps for nodal analysis

(1) Find the number of node in the circuit.

(2) Assume the voltage at each node.

(3) Connect the most common node to the ground. The voltage of this node is zero.

(4) Check whether any voltage source is connected with the ground if yes then the voltage of the node which is connected with the voltage source can be found directly.

(5) Now apply the KCL at all the remaining node by assuming the current in outward direction from the node.

(6) By solving the equations find the voltage at each node.

Example 1.13 Find the value of I shown in fig. 1.36 (a) using nodal analysis.

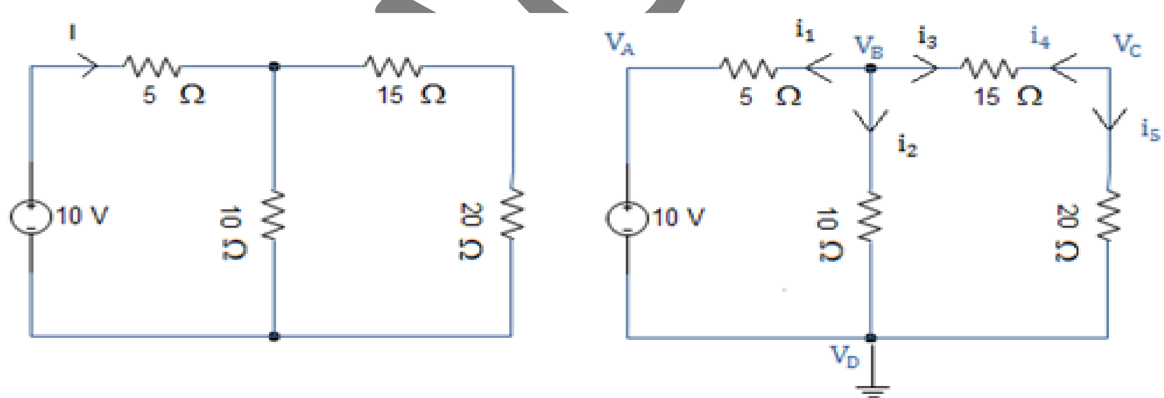


Fig. 1.36 (a) **Fig 1.36 (b)**

Solution: Apply the steps of the nodal analysis.

(1) The number of node in the circuit shown in fig. 1.36 (b) is 4 (A, B, C and D).

(2) Assume the voltage at the entire four nodes is (V_A, V_B, V_C and V_D) as shown in fig. 1.36 (b).

(3) The node D is the most common node so it is connected with the ground as shown in fig. 1.36 (b).

(4) The 10 V voltage source is directly connected with the ground. So the negative terminal voltage is zero and positive terminal voltage is 10 V so $V_A = 10\ V$

(5) Now apply the KCL at node B and C. At node B and C assume the current in the entire branch in outward direction as shown in fig. 1.36 (b).

At node B the equation of KCL is

$$i_1 + i_2 + i_3 = 0 \quad \text{.... (1.32)}$$

At node C the equation of KCL is

$$i_4 + i_5 = 0 \quad \text{.... (1.33)}$$

i_1 is flowing in 5 Ω resistance from node B to A so

$$i_1 = \frac{V_B - V_A}{5} = \frac{V_B - 10}{5}$$

i_2 is flowing in 10 Ω resistance from node B to D so

$$i_2 = \frac{V_B - V_D}{10} = \frac{V_B - 0}{10} = \frac{V_B}{10}$$

i_3 is flowing in 15 Ω resistance from node B to C so

$$i_3 = \frac{V_B - V_C}{15}$$

i_4 is flowing in 15 Ω resistance from node C to B so

$$i_4 = \frac{V_C - V_B}{15}$$

i_5 is flowing in 20 Ω resistance from node C to D so

$$i_5 = \frac{V_C - V_D}{20} = \frac{V_C - 0}{20} = \frac{V_C}{20}$$

Substitute the value of i_1, i_2, i_3, i_4 and i_5 in eq. (1.32) and (1.33) then

$$\frac{V_B - 10}{5} + \frac{V_B}{10} + \frac{V_B - V_C}{15} = 0 \qquad \frac{V_C - V_B}{15} + \frac{V_C}{20} = 0$$

$$11V_B - 2V_C = 60 \quad \text{....(1.34)} \qquad -4V_B + 7V_C = 0 \quad \text{....(1.35)}$$

By solving eq. (1.34) and (1.35), we get $V_B = 6.09\ V \quad V_C = 3.48\ V$

The value of I is

$$I = \frac{V_A - V_B}{5} = \frac{10 - 6.09}{5} = 0.782\ \text{Amp. } \mathbf{Ans.}$$

Example 1.14 Find the value of I shown in fig. 1.37 (a) using nodal analysis.

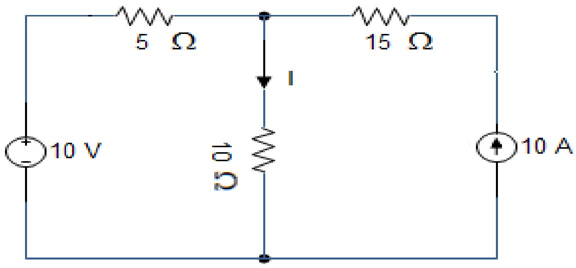


Fig. 1.37 (a)

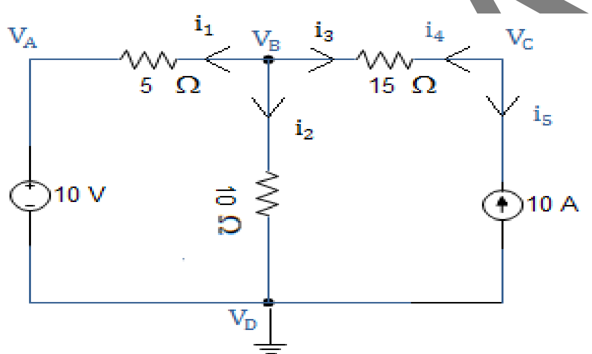


Fig 1.37 (b)

Solution: Apply the steps of the nodal analysis.

(1) The number of node in the circuit shown in fig. 1.37 (a) is 4 (A, B, C and D).

(2) Assume the voltage at the entire four nodes is (V_A, V_B, V_C and V_D) as shown in fig. 1.37 (b).

(3) The node D is the most common node so it is connected with the ground as shown in fig. 1.37 (b).

(4) The 10 V voltage source is directly connected with the ground. So the negative terminal voltage is zero and positive terminal voltage is 10 V so $V_A = 10\ V$

(5) Now apply the KCL at node B and C. At node B and C assume the current in the entire branch in outward direction.

At node B the equation of KCL is

$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_B - 10}{5} + \frac{V_B}{10} + \frac{V_B - V_C}{15} = 0$$

$$11V_B - 2V_C = 60$$

....(1.36)

At node C the equation of KCL is

$$i_4 + i_5 = 0$$

$$\frac{V_C - V_B}{15} - 10 = 0$$

$$-V_B + V_C = 150$$

....(1.37)

By solving eq. (1.36) and (1.37), we get $V_B = 40\ V \quad V_C = 190\ V$

The value of I is

$$I = \frac{V_B - V_D}{10} = \frac{40 - 0}{20} = 2 \text{ Amp. } \textbf{Ans.}$$

Example 1.15 Find the value of I shown in fig. 1.38 (a) using nodal analysis.

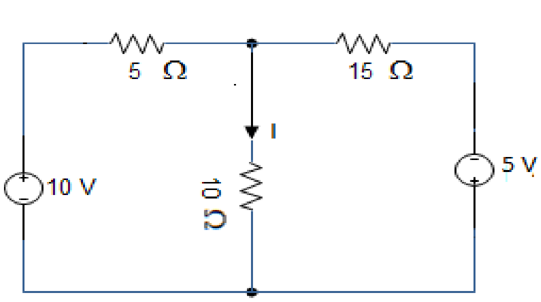


Fig. 1.38 (a)

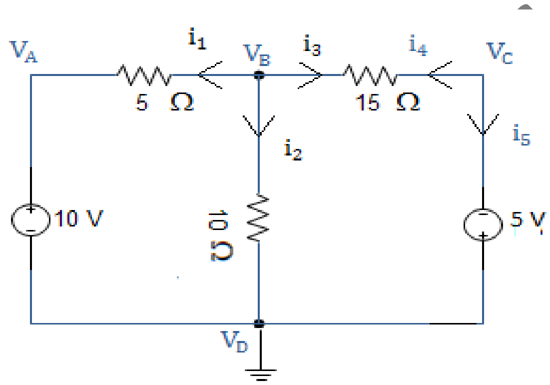


Fig. 1.38 (b)

Solution: Apply the steps of the nodal analysis.

(1) The number of node in the circuit shown in fig. 1.38 (a) is 4 (A, B, C and D).

(2) assume the voltage at all the four nodes is (V_A, V_B, V_C and V_D) as shown in fig. 1.38 (b).

(3) The node D is the most common node so it is connected with the ground as shown in fig. 1.38 (b).

(4) The 10 V and 5 V voltage source is directly connected with the ground. $V_A = 10\text{ V}, \; V_C = -5\text{ V}$

(5) Now apply the KCL at node B. At node B assume the current in the entire branch in outward direction.

At node B the equation of KCL is

$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_B - 10}{5} + \frac{V_B}{10} + \frac{V_B + 5}{15} = 0$$

$$11V_B = 50$$

$$V_B = 4.54\text{ V}$$

The value of I is

$$I = \frac{V_B - V_D}{10} = \frac{4.54 - 0}{10} = 0.454 \text{ Amp. } \textbf{Ans.}$$

Example 1.16 Find the value of I shown in fig. 1.39 (a) using nodal analysis.

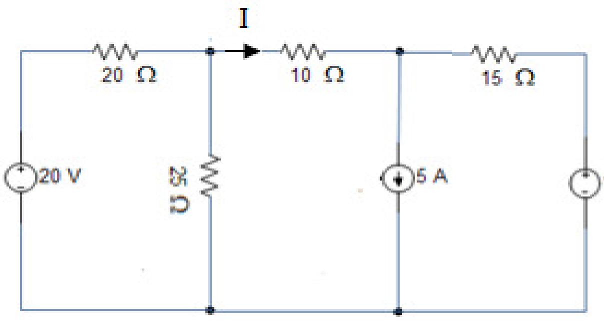


Fig. 1.39 (a)

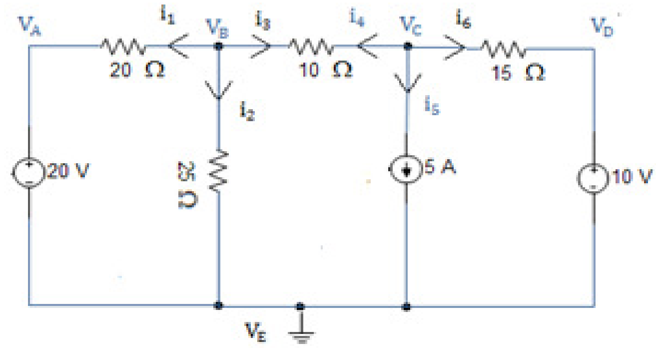


Fig. 1.39 (b)

Solution: Apply the steps of the nodal analysis.

The number of node in the circuit shown in fig. 1.2 is 5 (A,B,C,D and E)

Assume the voltage at all the four node is (V_A, V_B, V_C, V_D and V_E) as shown in fig.

The node E is the most common node so it is connected with the ground.

The voltage at node A and D is $V_A = 20\ \text{V}\ , V_D = 10\ \text{V}$

Now apply the KCL at node B and C. At node B and C assume the current in the entire branch in outward direction.

At node B the equation of KCL is

$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_B - 20}{20} + \frac{V_B}{25} + \frac{V_B - V_C}{10} = 0$$

$$19V_B - 10V_C = 100 \qquad \text{....(1.38)}$$

At node C the equation of KCL is

$$i_4 + i_5 + i_6 = 0$$

$$\frac{V_C - V_B}{10} + 5 + \frac{V_C - 10}{15} = 0$$

$$-3V_B + 5V_C = -150 \qquad \text{....(1.39)}$$

By solving eq. (1,38) and (1.39), we get $V_B = -15.38\ \text{V} \quad V_C = -39.23\ \text{V}$

The value of I is

$$I = \frac{V_B - V_C}{10} = \frac{-15.38 - (-39.23)}{10} = \frac{-15.38 + 39.23}{10} = 2.385\ \text{Amp.}\ \ \textbf{Ans.}$$

Example 1.17 Find the current in each resistance shown in fig. 1.40 (a) using nodal analysis.

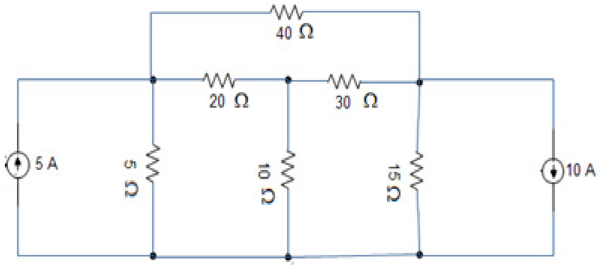


Fig. 1.40 (a)

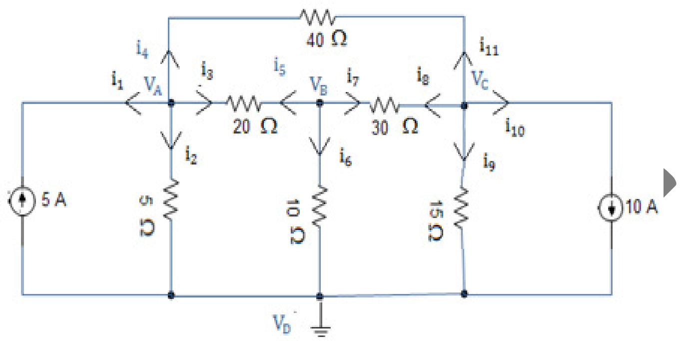


Fig. 1.40 (b)

Solution: Apply the steps of the nodal analysis.

(1) The number of node in the circuit shown in fig. 1.40 (a) is 4 (A, B, C and D).

(2) Assume the voltage at the entire four nodes is (V_A, V_B, V_C and V_D) as shown in fig. 1.40 (b).

(3) The node D is the most common node so it is connected with the ground as shown in fig. 1.40 (b).

(4) Now apply the KCL at node A, B and C. At node A, B and C assume the current in the entire branch in outward direction.

At node A the equation of KCL is

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$-5 + \frac{V_A - 0}{5} + \frac{V_A - V_B}{20} + \frac{V_A - V_C}{40} = 0$$

$$11V_A - 2V_B - V_C = 200$$

....(1.40)

At node B the equation of KCL is

$$i_5 + i_6 + i_7 = 0$$

$$\frac{V_B - V_A}{20} + \frac{V_B - 0}{10} + \frac{V_B - V_C}{30} = 0$$

$$-3V_A + 11V_B - 2V_C = 0$$

....(1.41)

At node C the equation of KCL is

$$i_8 + i_9 + i_{10} + i_{11} = 0$$

$$\frac{V_C - V_B}{30} + \frac{V_C - 0}{15} + 10 + \frac{V_C - V_A}{40} = 0$$

$$-3V_A - 4V_B + 15V_C = -1200$$

....(1.42)

By solving eq. (1.40), (1.41) and (1.42), we get $V_A = 8.48\ \text{V} \quad V_B = -12.53\ \text{V} \quad V_C = -81.64\ \text{V}$

The value of current in

$$5\ \Omega\ \text{resistance} = \frac{V_A - 0}{5} = \frac{8.48 - 0}{5} = 1.67\ \text{Amp.}\ \ \textbf{Ans}.$$

$$10\ \Omega\ \text{resistance} = \frac{V_B - 0}{10} = \frac{-12.53 - 0}{10} = -1.253\ \text{Amp.}\ \ \textbf{Ans}.$$

$$15\ \Omega\ \text{resistance} = \frac{V_C - 0}{15} = \frac{-81.64 - 0}{15} = -5.44\ \text{Amp.}\ \ \textbf{Ans}.$$

$$20\ \Omega\ \text{resistance} = \frac{V_A - V_B}{20} = \frac{8.48 - (-12.53)}{20} = \frac{8.48 + 12.53}{20} = 1.05\ \text{Amp.}\ \ \textbf{Ans}.$$

$$30\ \Omega\ \text{resistance} = \frac{V_B - V_C}{30} = \frac{-12.53 - (-81.64)}{30} = \frac{-12.53 + 81.64}{30} = 2.30\ \text{Amp.}\ \ \textbf{Ans}.$$

$$40\ \Omega\ \text{resistance} = \frac{V_A - V_C}{40} = \frac{8.48 - (-81.64)}{40} = \frac{8.48 + 81.64}{40} = 2.25\ \text{Amp.}\ \textbf{Ans}.$$