UNIT 2

AC FUNDAMENTAL

2.1 AC FUNDAMENTAL

A voltage of current is called alternating if it change periodically in direction and continuously change magnitude.

Waveform: The graph of the voltage or current when plotted against time is called the waveform.

Cycle: When an alternating wave goes through one complete set of positive and negative value it is called one cycle

Time period: Time taken to complete one cycle is called time period. It is denoted by T and unit is sec.

Frequency: The number of cycle completed in one second is called the frequency. It is denoted by f and unit is Hz.

$$f = \frac{1}{T}$$

Angular velocity or frequency:

$$\omega = 2\pi f = \frac{1}{2\pi T}$$

Symmetrical waveform: An alternating waveform is called symmetrical if the area of positive and negative part is equal. The waveform shown in fig. 2.1 is symmetrical.

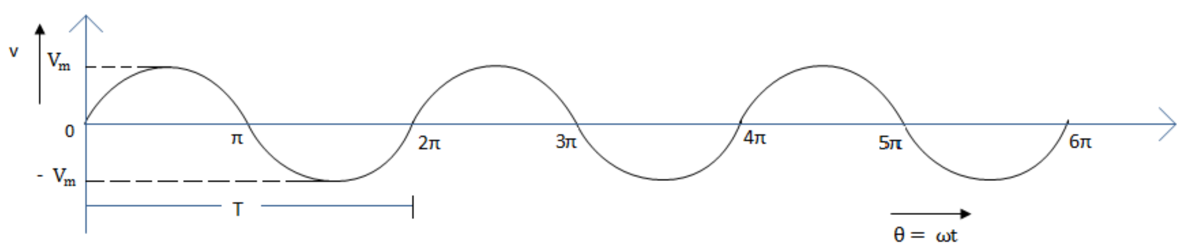

Fig. 2.1

Unsymmetrical waveform: An alternating waveform is called unsymmetrical if the area of positive and negative part is unequal. The waveform shown in fig. 2.2 is unsymmetrical.

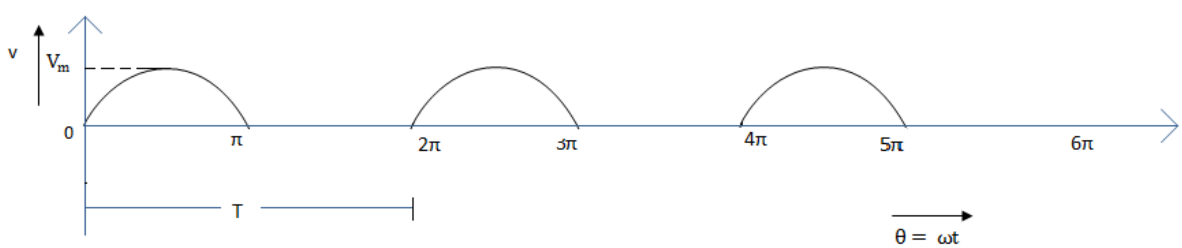

Fig. 2.2

2.2 DIFFERENT TYPES OF WAVEFORM

2.2.1 **Sinusoidal waveform:** The sinusoidal waveform is shown in fig. 2.3.

The waveform is symmetrical.

The one cycle of the waveform is 0 to 2π.

The equation represent the sinusoidal waveform for one cycle is,

$$v = V_m \sin\theta \qquad \text{When } 0 < \theta < 2\pi$$

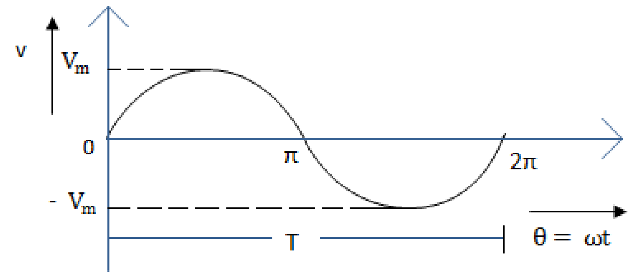

Fig. 2.3

2.2.2 **Half wave rectifier waveform:** The half wave rectifier waveform is shown in fig. 2.4.

The waveform is unsymmetrical.

The one cycle of the waveform is 0 to 2π.

The equation represent the half wave rectifier waveform for one cycle is,

$$v = V_m \sin\theta \qquad \text{When } 0 < \theta < \pi$$

$$= 0 \qquad \text{When } \pi < \theta < 2\pi$$

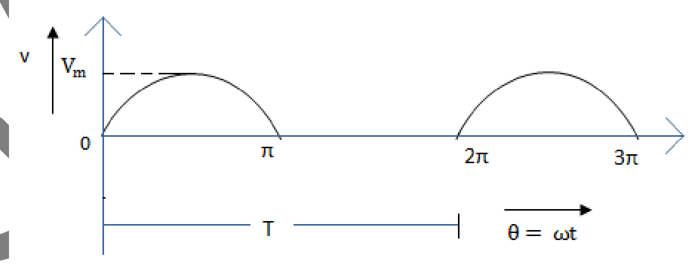

Fig. 2.4

2.2.3 **Full wave rectifier waveform:** The full wave rectifier waveform is shown in fig. 2.5.

The waveform is unsymmetrical.

The one cycle of the waveform is 0 to π.

The equation represent the full wave rectifier waveform for one cycle is,

$$v = V_m \sin\theta \qquad \text{When } 0 < \theta < \pi$$

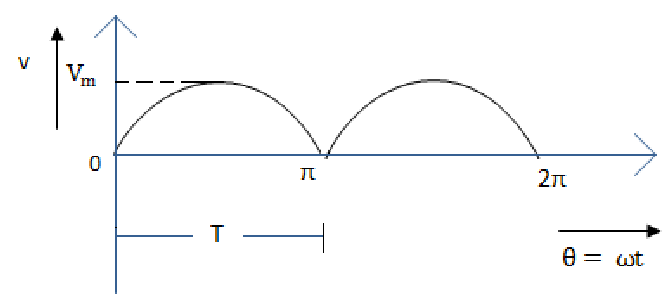

Fig. 2.5

2.2.4 **Square waveform:** The square waveform is shown in fig. 2.6.

The waveform is symmetrical.

The one cycle of the waveform is 0 to 2π.

The equation represent the square waveform for one cycle is,

$$v = V_m \qquad \text{When } 0 < \theta < \pi$$
$$= -V_m \qquad \text{When } \pi < \theta < 2\pi$$

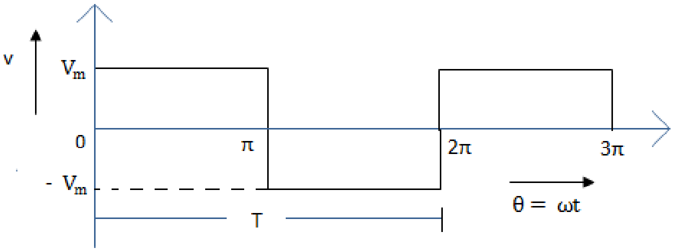

Fig. 2.6

2.2.5 **Triangular waveform:** The triangular waveform is shown in fig. 2.7.

The waveform is symmetrical.

The one cycle of the waveform is 0 to 2π.

The equation represent the sinusoidal waveform for one cycle is,

$$v = \frac{2V_m\theta}{\pi} \qquad \text{When } 0 < \theta < \pi/2$$

$$v = -\frac{2V_m\theta}{\pi} + 2V_m \qquad \text{When } \pi/2 < \theta < \pi$$

$$v = -\frac{2V_m\theta}{\pi} + 2V_m \qquad \text{When } \pi < \theta < 3\pi/2$$

$$v = \frac{2V_m\theta}{\pi} - 4V_m \qquad \text{When } 3\pi/2 < \theta < 2\pi$$

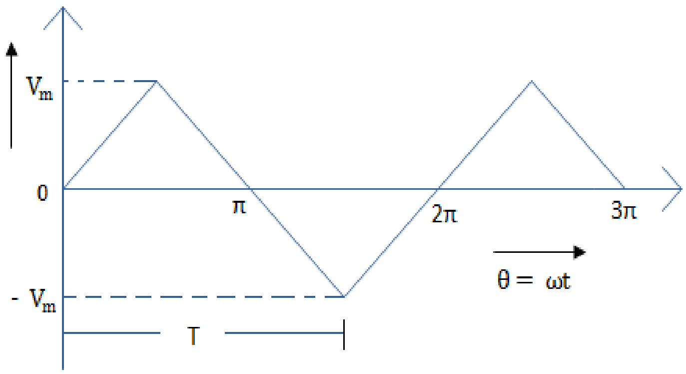

Fig. 2.7

2.3 VALUES OF ALTERNATING QUANTITY

In AC system the magnitude of voltage and current change continuously so the problem is how to specify the alternating voltage or current. An alternating voltage or current can be expressed by different value.

2.3.1 **Instantaneous value:** The value of alternating quantity at any particular time is called instantaneous value. It is denoted by small letter (v or i).

2.3.2 **Amplitude (Peak value):** The maximum value of alternating quantity is called amplitude. It is denoted by V_m or I_m.

Example 2.1 An alternating current is given by $i = 40 \sin 600t$. Find (1) Frequency (2) Time period (3) Maximum value (4) The time taken for the current to reach 20 Amp. (5) Find the value of current when time is 0.5 msec.

Solution: The given eq. of current is $i = 40 \sin 600t$

By compare the given eq. by the standard eq. $i = I_m \sin \omega t$, $I_m = 40$ Amp. $\omega = 600$ Radian

(1) Frequency (f): We know ω = 2πf

$$f = \frac{\omega}{2\pi} = \frac{600}{2\pi} = 95.5 \text{ Hz } \textbf{Ans}.$$

(2) Time period (T): We know

$$T = \frac{1}{f} = \frac{1}{95.5} = 0.0105 \text{ Sec } \textbf{Ans}.$$

(3) Maximum va0lue (I_m): We know

$$I_m = 40 \text{ Amp. } \textbf{Ans}.$$

(4) The time taken for the current to reach 20 Amp: Given I = 20 Amp.

$$20 = 40 \sin 600t$$

$$t = \frac{1}{600} \sin^{-1} \frac{20}{40}$$

Solve the eq. in radian

$$t = 8.7 \times 10^{-4} = 0.87 \text{ mSec } \textbf{Ans}.$$

(5) Find the value of current when time is 0.5 msec: Given t = 0.5 msec.

$$i = 40 \sin(600 \times 0.5 \times 10^{-3})$$

Solve the eq. in radian

$$i = 11.82 \text{ Amp } \textbf{Ans}.$$

Example 2.2 A sinusoidal varying alternating current of frequency 60 Hz has a maximum value of 50 Amp. Find (1) the equation of instantaneous value (2) Find the time period. (3) Angular frequency (4) the time taken for the current to reach 10 Amp. (5) Find the value of current when time is 1/250 sec.

Solution: Given f = 60 Hz. $I_m = 50$ Amp.

(1) The standard eq. of sinusoidal current is $i = I_m \sin \omega t$

$$\omega = 2\pi f = 2\pi \times 60 = 377 \text{ radian}$$

$$i = 50 \sin 377t \textbf{ Ans}.$$

(2) Time period (T): We know

$$T = \frac{1}{f} = \frac{1}{60} = 0.0167 = 16.7 \text{ mSec } \textbf{Ans}.$$

(3) Angular frequency (ω): $\omega = 377 \text{ radian } \textbf{Ans}.$

(4) The time taken for the current to reach 10 Amp.: Given i = 10 Amp.

$$10 = 50\sin 377t$$
$$t = \frac{1}{377}\sin^{-1}\frac{10}{50}$$

Solve the eq. in radian

$$t = 5.34 \times 10^{-4} = 0.534 \text{ mSec } \mathbf{Ans}.$$

(5) Find the value of current when time is 1/250 sec: Given t = 1/250 sec.

$$i = 40\sin\left(\frac{377}{250}\right)$$

Solve the eq. in radian

$$i = 39.92 \text{ Amp } \mathbf{Ans}.$$

2.3.3 **Average Value:** The average value of AC is equal to that value of DC which transfers same charge in same time in a circuit which is transfer by AC. It is denoted by V_{av} or I_{av} .

$$\text{Average value} = \frac{\text{Area of one cycle}}{\text{Base of one cycle}}$$

For symmetrical wave area of one cycle is always zero.

For symmetrical wave

$$\text{Average value} = \frac{\text{Area of half cycle}}{\text{Base of half cycle}} = \frac{\int_0^{T/2}(\text{v or i})d\theta}{T/2}$$

Where v or i is the equation of AC from 0 to T/2, T is the time period.

For Unsymmetrical wave

$$\text{Average value} = \frac{\text{Area of full cycle}}{\text{Base of full cycle}} = \frac{\int_0^{T}(\text{v or i})d\theta}{T}$$

Where v or i is the equation of AC from 0 to T, T is the time period.

2.3.4 **RMS Value:** The RMS value of AC is equal to that value of DC which produce the same amount of heat when flowing the same time period in a resistance which produce by AC. It is denoted by V or I.

$$\text{RMS value} = \sqrt{\frac{\int_0^{T}(\text{v or i})^2 d\theta}{T}}$$

Where v or i is the equation of AC from 0 to T, T is the time period.

<u>**Note:**</u> For symmetrical wave the RMS value for half cycle and full cycle will be same.

2.4 FORM FACTOR AND PEAK FACTOR

2.4.1 **Form Factor:** It is the ratio of RMS value to Average value.

$$\text{Form Factor(FF)} = \frac{\text{RMS value}}{\text{Average value}}$$

2.4.2 **Peak Factor:** It is the ratio of Maximum value to RMS value.

$$\text{Peak Factor(PF)} = \frac{\text{Maximum value}}{\text{RMS value}}$$

Example 2.3 Find the Average value, RMS value, Form Factor and Peak Factor for sinusoidal wave.

Solution: The sinusoidal wave form is shown in fig. 2.8.

The waveform is symmetrical.

The one cycle of the waveform is 0 to 2π, T = 2π.

The equation represent the sinusoidal waveform for one cycle is,

$$v = V_m \sin\theta \qquad \text{When } 0 < \theta < 2\pi$$

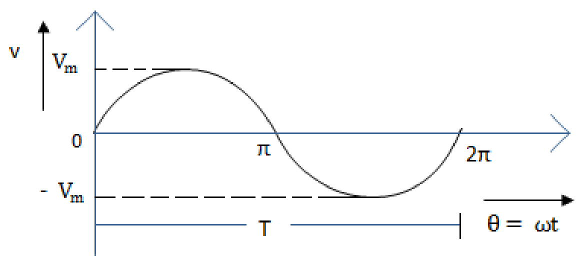

Fig. 2.8

(1) Average Value: For symmetrical wave

$$\text{Average value} = \frac{\text{Area of half cycle}}{\text{Base of half cycle}} = \frac{\int_0^{\frac{T}{2}} v d\theta}{\frac{T}{2}} = \frac{\int_0^{\pi} V_m \sin\theta \, d\theta}{\pi} = \frac{V_m(-\cos\theta)_0^{\pi}}{\pi}$$

$$= \frac{V_m(-\cos\pi + \cos 0)}{\pi} = \frac{V_m(1+1)}{\pi} = \frac{2V_m}{\pi} = 0.637V_m$$

$$\text{Average value} = 63.7\% \text{ of } V_m \ \textbf{Ans}.$$

The average value of sinusoidal wave is 63.7% of maximum value.

(2) RMS value:

$$\text{RMS value} = \sqrt{\frac{\int_0^T v^2 d\theta}{T}} = \sqrt{\frac{\int_0^{2\pi} (V_m \sin\theta)^2 d\theta}{2\pi}} = \sqrt{\frac{V_m^2 \int_0^{2\pi} \sin^2\theta d\theta}{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi} \int_0^{2\pi} \frac{(1-\cos 2\theta) d\theta}{2}} = \sqrt{\frac{V_m^2}{4\pi} \left(\theta - \frac{\sin 2\theta}{2}\right)_0^{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{4\pi} \left(2\pi - \frac{\sin 4\pi}{2} - 0 + \frac{\sin 0}{2}\right)} = \sqrt{\frac{V_m^2 (2\pi)}{4\pi}} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}} = 0.707V_m$$

$$\text{RMS VALUE} = 70.7\ \%\ \text{of}\ V_m\ \textbf{Ans.}$$

The RMS value of sinusoidal wave is 70.7% of maximum value.

(3)Form Factor:

$$\text{Form Factor(FF)} = \frac{\text{RMS value}}{\text{Average value}} = \frac{0.707V_m}{0.637V_m} = 1.11\ \textbf{Ans.}$$

(4) Peak Factor:

$$\text{Peak Factor(PF)} = \frac{\text{Maximum value}}{\text{RMS value}} = \frac{V_m}{0.707V_m} = 1.414\ \textbf{Ans.}$$

Example 2.4: Find the Average value, RMS value, Form Factor and Peak Factor for half wave rectifier.

Solution: The Half wave rectifier waveform is shown in fig. 2.9.

The waveform is unsymmetrical.

The one cycle of the waveform is 0 to 2π, T = 2π.

The equation represent the half wave rectifier waveform for one cycle is,

$$v = V_m \sin\theta \qquad \text{When } 0 < \theta < \pi$$

$$= 0 \qquad \text{When } \pi < \theta < 2\pi$$

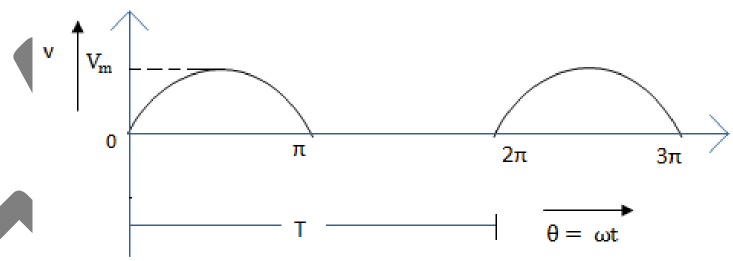

Fig. 2.9

(1) Average Value: For unsymmetrical wave

$$\text{Average value} = \frac{\text{Area of full cycle}}{\text{Base of full cycle}} = \frac{\int_0^T v d\theta}{T} = \frac{\int_0^{2\pi} v d\theta}{2\pi} = \frac{\int_0^{\pi} V_m \sin\theta\, d\theta + \int_{\pi}^{2\pi} 0 d\theta}{2\pi} = \frac{V_m(-\cos\theta)_0^{\pi}}{2\pi}$$

$$= \frac{V_m(-\cos\pi + \cos 0)}{2\pi} = \frac{V_m(1+1)}{2\pi} = \frac{2V_m}{2\pi} = \frac{V_m}{\pi} = 0.318V_m$$

$$\text{Average value} = 31.8\%\ \text{of}\ V_m\ \textbf{Ans.}$$

The average value of half wave rectifier is 31.8% of maximum value.

(2) RMS value:

$$\text{RMS value} = \sqrt{\frac{\int_0^T v^2 d\theta}{T}} = \sqrt{\frac{\int_0^{\pi} v^2 d\theta + \int_{\pi}^{2\pi} v^2 d\theta}{2\pi}} = \sqrt{\frac{\int_0^{\pi}(V_m \sin\theta)^2 d\theta + \int_{\pi}^{2\pi} 0 d\theta}{2\pi}} = \sqrt{\frac{V_m^2 \int_0^{\pi} \sin^2\theta d\theta}{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\frac{(1-\cos 2\theta)d\theta}{2}} = \sqrt{\frac{V_m^2}{4\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}} = \sqrt{\frac{V_m^2}{4\pi}\left(\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right)}$$

$$\text{RMS value} = \sqrt{\frac{V_m^2(\pi)}{4\pi}} = \sqrt{\frac{V_m^2}{4}} = \frac{V_m}{2} = 0.5V_m$$

$\text{RMS value} = 50\%\ \text{of}\ V_m$ **Ans.**

The RMS value of half wave rectifier is 50% of maximum value.

(3) Form Factor:

$$\text{Form Factor(FF)} = \frac{\text{RMS value}}{\text{Average value}} = \frac{0.5\ V_m}{0.318\ V_m} = 1.57\ \textbf{Ans.}$$

(4) Peak Factor:

$$\text{Peak Factor(PF)} = \frac{\text{Maximum value}}{\text{RMS value}} = \frac{V_m}{0.5\ V_m} = 2\ \textbf{Ans.}$$

Example 2.5: Find the Average value, RMS value, Form Factor and Peak Factor for square wave.

Solution: The square waveform is shown in fig. 2.10.

The waveform is symmetrical.

The one cycle of the waveform is 0 to 2π, T = 2π.

The equation represent the square waveform for one cycle is,

$v = V_m$ When $0 < \theta < \pi$

$= -V_m$ When $\pi < \theta < 2\pi$

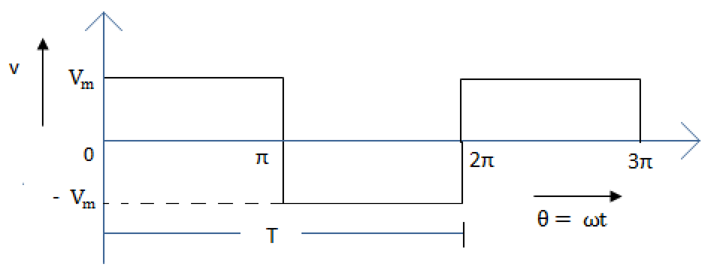

Fig. 2.10

(1) Average Value: For symmetrical wave

$$\text{Average value} = \frac{\text{Area of half cycle}}{\text{Base of half cycle}} = \frac{\int_0^{T/2} v d\theta}{T/2} = \frac{\int_0^{\pi} v d\theta}{\pi} = \frac{\int_0^{\pi} V_m d\theta}{\pi} = \frac{V_m(\theta)_0^{\pi}}{\pi} = \frac{V_m(\pi - 0)}{\pi}$$

$$= \frac{V_m \pi}{\pi} = V_m$$

$\text{Average value} = V_m$ **Ans.**

The average value of square wave is equal to maximum value.

(2) RMS value:

$$\text{RMS value} = \sqrt{\frac{\int_0^T v^2 d\theta}{T}} = \sqrt{\frac{\int_0^{\pi} v^2 d\theta + \int_{\pi}^{2\pi} v^2 d\theta}{2\pi}} = \sqrt{\frac{\int_0^{\pi}(V_m)^2 d\theta + \int_{\pi}^{2\pi}(-V_m)^2 d\theta}{2\pi}}$$

$$= \sqrt{\frac{V_m^2[(\theta)_0^{\pi} + (\theta)_{\pi}^{2\pi}]}{2\pi}} = \sqrt{\frac{V_m^2[(\pi - 0) + (2\pi - \pi)]}{2\pi}} = \sqrt{\frac{V_m^2[\pi + \pi]}{2\pi}}$$

$$\text{RMS Value} = \sqrt{\frac{V_m^2(2\pi)}{4\pi}} = \sqrt{V_m^2} = V_m$$

$$\text{RMS value} = V_m \ \textbf{Ans.}$$

The RMS value of square wave is equal to maximum value.

(3) Form Factor:

$$\text{Form Factor(FF)} = \frac{\text{RMS value}}{\text{Average value}} = \frac{V_m}{V_m} = 1 \ \textbf{Ans.}$$

(4) Peak Factor:

$$\text{Peak Factor(PF)} = \frac{\text{Maximum value}}{\text{RMS value}} = \frac{V_m}{V_m} = 1 \ \textbf{Ans.}$$

Example 2.6: Find the Average value, RMS value, Form Factor and Peak Factor for square wave.

Solution: The triangular waveform is shown in fig. 2.11.

The waveform is symmetrical. This waveform is also symmetrical between 0 to π. So we can find the average and RMS value between 0 to π/2.

The equation represents the triangular waveform from 0 to π/2

$$v = \frac{2V_m\theta}{\pi} \qquad \text{When } 0 < \theta < \pi/2$$

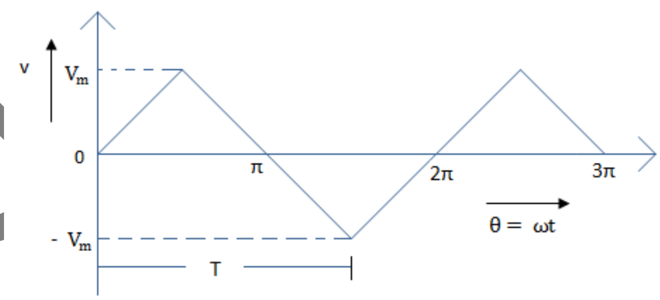

Fig. 2.11

(1) Average Value:

$$\text{Average value} == \frac{\int_0^{\pi/2} v\, d\theta}{\pi/2} = \frac{\int_0^{\pi/2}\left(\frac{2V_m\theta}{\pi}\right) d\theta}{\pi/2} = \frac{4V_m\int_0^{\pi/2}\theta\, d\theta}{\pi^2} = \frac{4V_m}{\pi^2}\left(\frac{\theta^2}{2}\right)_0^{\pi/2} = \frac{2V_m}{\pi^2}\left(\frac{\pi^2}{4}\right) = \frac{V_m}{2}$$

$$= 0.5V_m$$

$$\text{Average value} = 50\% \text{ of } V_m \ \textbf{Ans.}$$

The average value of triangular wave is 50 % of maximum value.

(2) RMS value:

$$\text{RMS value} = \sqrt{\frac{\int_0^{\frac{\pi}{2}} v^2 d\theta}{\frac{\pi}{2}}} = \sqrt{\frac{\int_0^{\frac{\pi}{2}}\left(\frac{2V_m\theta}{\pi}\right)^2 d\theta}{\frac{\pi}{2}}} = \sqrt{\frac{8V_m^2\int_0^{\frac{\pi}{2}}\theta^2\, d\theta}{\pi^3}} = \sqrt{\frac{8V_m^2}{\pi^3}\left(\frac{\theta^3}{3}\right)_0^{\frac{\pi}{2}}}$$

$$\text{RMS value} = \sqrt{\frac{8V_m^2}{3\pi^3}\left(\frac{\pi^3}{8}\right)} = \sqrt{\frac{V_m^2}{3}} = \frac{V_m}{\sqrt{3}} = 0.5773V_m$$

$$\text{RMS value} = 57.73\% \text{ of } V_m \textbf{ Ans.}$$

The RMS value of triangular wave is 57.73 % of maximum value.

(3) Form Factor:

$$\text{Form Factor(FF)} = \frac{\text{RMS value}}{\text{Average value}} = \frac{0.5773V_m}{0.5\ V_m} = 1.15 \textbf{ Ans.}$$

(4) Peak Factor:

$$\text{Peak Factor(PF)} = \frac{\text{Maximum value}}{\text{RMS value}} = \frac{V_m}{0.5773V_m} = 1.37 \textbf{ Ans.}$$

2.4 PHASOR DIARGAM

Concept of phasor: The representation of alternating quantity by a line of definite length rotating in anticlockwise direction with angle is called the phasor.

Phase difference: When two alternating quantity are passing through a particular point at the different instant. Then these two quantities have a phase difference.

Phasor representation of sinusoidal varying voltage and current:

Case 1(same phase): When voltage and current are passing through a point at same instant then they are called in same phase. It can be represented by eq. (2.1) and (2.2). The waveform and phasor diagram of this condition is shown in fig. 2.12 (a) and 2.12 (b) respectively.

$$v = V_m \sin\theta \quad ....(2.1)$$

$$i = I_m \sin\theta \quad ....(2.2)$$

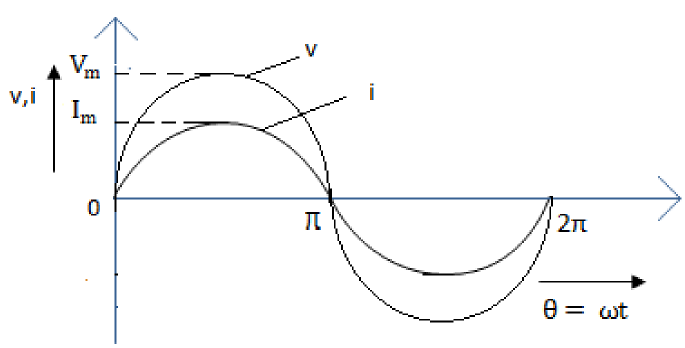

Fig.2.12 (a)

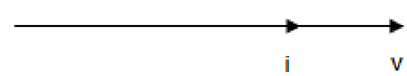

Fig. 2.12 (b)

Case 2(lagging): When current is passing through a point after voltage is passing then it is called current is lagging from voltage. It can be represented by eq. (2.3) and (2.4). The waveform and phasor diagram of this condition is shown in fig. 2.13 (a) and 2.13 (b) respectively.

$$v = V_m \sin\theta \quad \text{....(2.3)}$$

$$i = I_m \sin(\theta - \emptyset) \quad \text{....(2.4)}$$

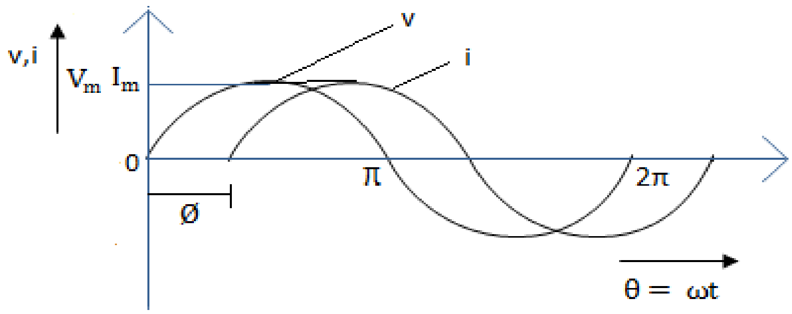

Fig.2.13 (a)

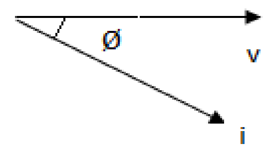

Fig. 2.13 (b)

Case 3(leading): When current is passing through a point before voltage is passing then it is called current is leading from voltage. It can be represented by eq. (2.5) and (2.6). The waveform and phasor diagram of this condition is shown in fig. 2.14 (a) and 2.14 (b) respectively.

$$v = V_m \sin\theta \quad \text{....(2.5)}$$

$$i = I_m \sin(\theta + \emptyset) \quad \text{....(2.6)}$$

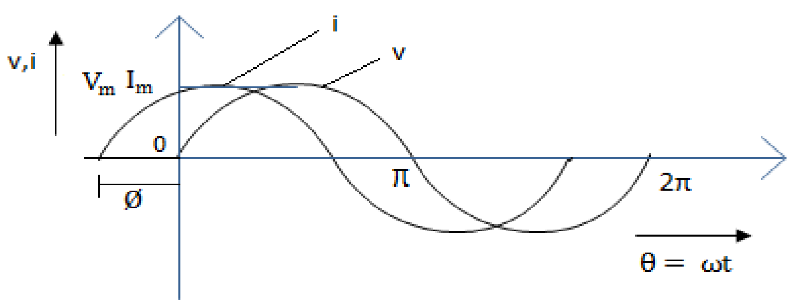

Fig.2.14 (a)

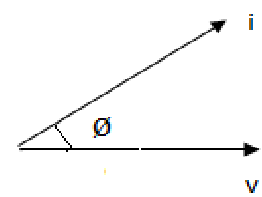

Fig. 2.14 (b)

2.5 ADDITION OF PHASOR QUANTITY

The phasor quantity can be added using component method. Steps for component method are

(1) Draw the phasor diagram showing all the alternating quantity
(2) Resolve all the quantity along X and Y axis.
(3) Find the total X component and Y component
(4) Maximum value of resultant $(V_m) = \sqrt{(\text{X component})^2 + (\text{Y component})^2}$

(5) Phase angle of resultant $(\theta) = \tan^{-1}\left(\frac{\text{Y component}}{\text{X component}}\right)$

(6) The resultant can be written as $v_r = V_m \sin(\omega t + \theta)$

Example 2.7 Find the resultant of the alternating quantities,

$$v_1 = 100\sin(\omega t) \qquad v_2 = 50\sin(\omega t - 60°)$$

Solution: The phasor diagram for maximum value of v_1 and v_2 is shown in fig. 2.15.

$\text{X xomponent} = 100\cos 0° + 50\cos 60° = 125$

$\text{Y xomponent} = 100\sin 0° - 50\sin 60° = -43.3$

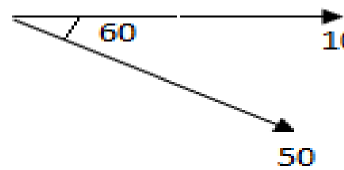

Fig. 2.15

$$V_m = \sqrt{(\text{X component})^2 + (\text{Y component})^2} = \sqrt{(125)^2 + (-43.3)^2} = 132.28$$

$$\theta = \tan^{-1}\left(\frac{\text{Y component}}{\text{X component}}\right) = \tan^{-1}\left(\frac{-43.3}{125}\right) = -19.10°$$

$$v_r = V_m \sin(\omega t + \theta) = 132.28\sin(\omega t - 19.10°)\ \textbf{Ans.}$$

Example 2.8 Find the resultant of the alternating quantities,

$$v_1 = 50\sin(\omega t) \qquad v_2 = 25\sin(\omega t - 45°) \qquad v_3 = 75\sin(\omega t + 30°) \qquad v_4 = -100\cos(\omega t)$$

Solution: The base of v_4 is different from the all the voltage. Before drawing the phasor diagram make all the alternating quantity of same base.

$$v_4 = -100\cos(\omega t) = -100\sin(90 - \omega t) = 100\sin(\omega t - 90°)$$

The phasor diagram for maximum value of all the alternating quantity is shown in fig. 2.16

$$\text{X component} = 50\cos 0° + 25\cos 45° + 75\cos 30° + 100\cos 90° = 132.68$$

$$\text{Y component} = 50\sin 0° - 25\sin 45° + 75\sin 30° - 100\sin 90° = -80.18$$

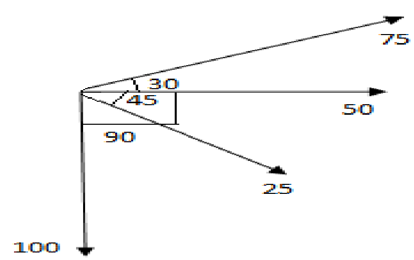

Fig. 2.16

$$V_m = \sqrt{(\text{X component})^2 + (\text{Y component})^2} = \sqrt{(132.68)^2 + (-80.18)^2} = 155$$

$$\theta = \tan^{-1}\left(\frac{\text{Y component}}{\text{X component}}\right) = \tan^{-1}\left(\frac{-80.18}{132.68}\right) = -31.14°$$

$$v_r = V_m \sin(\omega t + \theta) = 155\sin(\omega t - 31.14°)\ \textbf{Ans.}$$